Applications of Kalman Filtering to Real-time CO_2 and NH_3 Concentration Measurements

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Abstract

Kalman filtering techniques are applied to the real-time simultaneous detection of CO_2 and NH_3 using a portable diode laser-based sensor utilizing vibrational overtone direct-absorption spectroscopy at 1.53 μm. These filters aid in the practical real-time detection of small concentrations in the presence of noise and can improve the signal-to-noise ratio. The filter is dynamic and has a frequency response that is not fixed but can change based on the noise and signal statistics. Kalman filter theory is discussed and real world applications are demonstrated.

Motivation

Real-time concentration measurements with a Tunable Diode Laser spectrometer for use in a spacecraft environment

- Vapor concentration fluctuations occur on a much different time scale than the noise
- Example, a leaking ammonia cooling line on the International Space Station might take a few seconds to form, whereas most Johnson, quantum, and laser noises have components that change with a much faster time constant.
- Allows the use of a recursive time-series Kalman filter requiring little information be passed from measurement to measurement

Kalman Filtering

- Originally developed by R.E. Kalman for aerospace navigation applications in which noisy position and velocity observations were used to calculate a vehicle's estimated "state vector"
- Provides and efficient computational (recursive) solution of the least-squares method
- Allows for prediction even when the precise nature of the modeled system is unknown
- Kalman filter is optimal with respect all criteria
- Implemented entirely in Labview independent of sensor configuration

Modeling Equations

- *Concentration Model*

$$x_{k+1} = x_k + w_k \text{ , or in general } \quad x_{k+1} = x_k + u_k + w_k$$

where $E[w_k] = 0$, and white noise σ_w^2

- *Observation Model*

$$z_k = x_k + v_k \quad ,$$

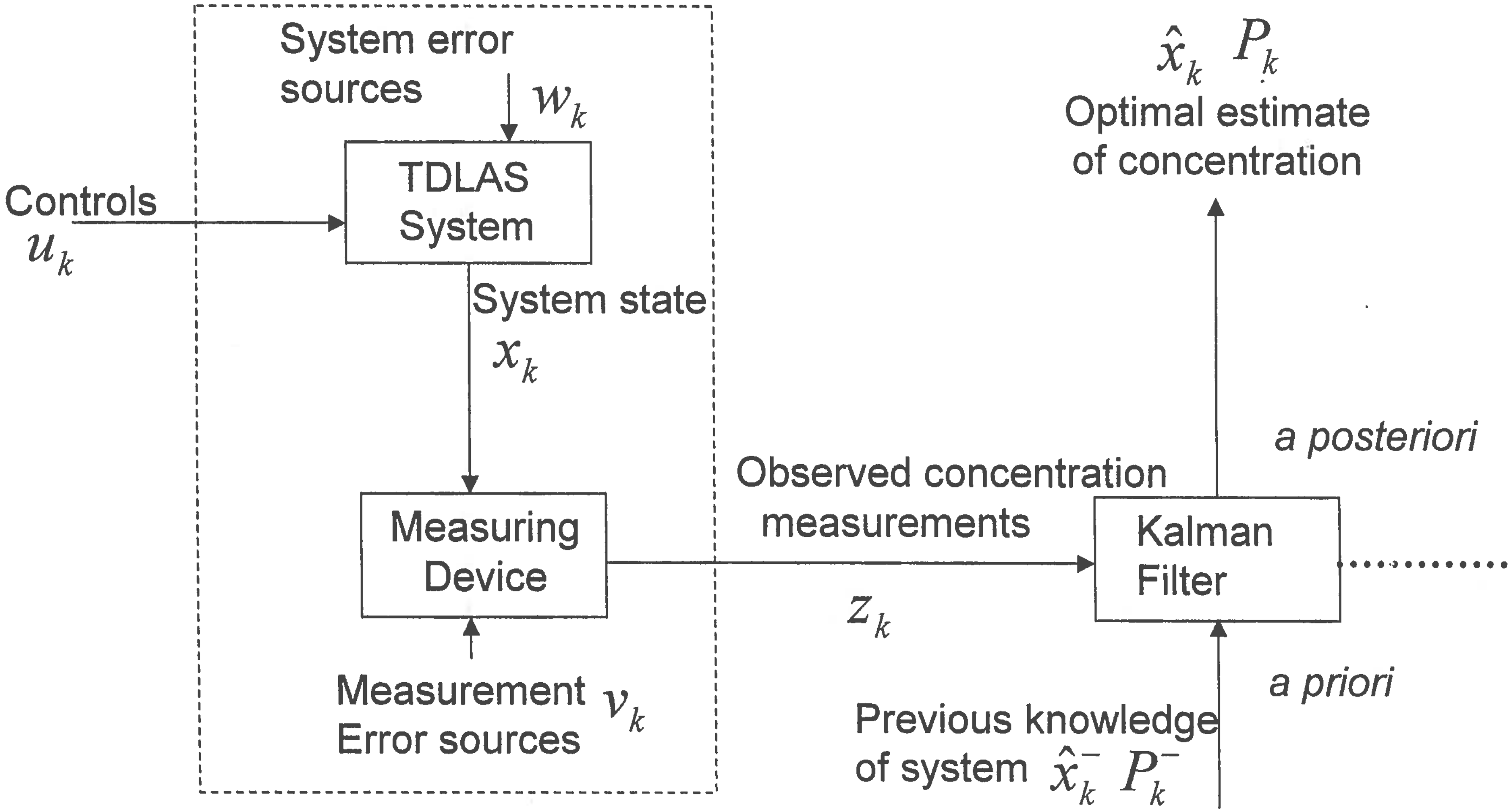
where $E[v_k] = 0$, and white noise σ_v^2

x_k is the "true" concentration at time k

z_k is the measured concentration at time k

w_k and v_k are uncorrelated random variables representing system error sources and measurement error sources respectively

Kalman Filter Application to TDLAS



Modeling Equations (Definitions)

$e_k^- \equiv x_k - \hat{x}_k^-$ *a priori* estimate error

$e_k \equiv x_k - \hat{x}_k$ *a posteriori* estimate error

x_k is the "true" concentration

$\hat{x}_k^-$ *a priori* concentration estimate

$\hat{x}_k$ *a posteriori* concentration estimate

This is what we want to minimize

$P_k^- = E[e_k^- e_k^-]$ *a priori* estimate error variance

$P_k = E[e_k e_k]$ *a posteriori* estimate error variance

Kalman Filter Equations

$$\underbrace{\hat{x}_k = \hat{x}_k^-}_{\text{Prediction}} + \underbrace{K_k}_{\text{Kalman Gain}} \underbrace{(z_k - \hat{x}_k^-)}_{\text{Residual}}$$, where

$$K_k = P_k^-(P_k^- + \sigma_v^2)^{-1}$$, Kalman Gain

$$P_{k+1}^- = P_k + \sigma_w^2$$, *a priori* estimate error variance for next interation

$$P_k = (1 - K_k)P_k^-$$, *a posteriori* estimate error variance

Kalman Filter Operation

$\hat{x}_{k+1}^{-}$ & P_{k+1}^{-} becomes the new $\hat{x}_{k}^{-}$ & P_{k}^{-}

Initial Estimates for $\hat{x}_{k}^{-}$ and P_{k}^{-}

Time Update ("Predict")

(1) Project the Concentration Ahead

$$\hat{x}_{k+1}^{-} = \hat{x}_{k}$$

(2) Project the Error Uncertainty

$$P_{k+1}^{-} = P_{k} + \sigma_{v}^{2}$$

Measurement Update ("Correct")

(1) Compute the Kalman Gain

$$K_{k} = P_{k}^{-}(P_{k}^{-} + \sigma_{w}^{2})^{-1}$$

(2) Update concentration estimate with measurement

$$\hat{x}_{k} = \hat{x}_{k}^{-} + K_{k}\left(z_{k} - \hat{x}_{k}^{-}\right)$$

(3) Update the error uncertainty

$$P_{k} = P_{k}^{-}(1 - K_{k})$$

$\hat{x}_{k}$ P_{k}

Filter Parameter Tuning

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - \hat{x}_k^-)$$

$$K_k = \frac{P_k^-}{(P_k^- + \sigma_v^2)}$$

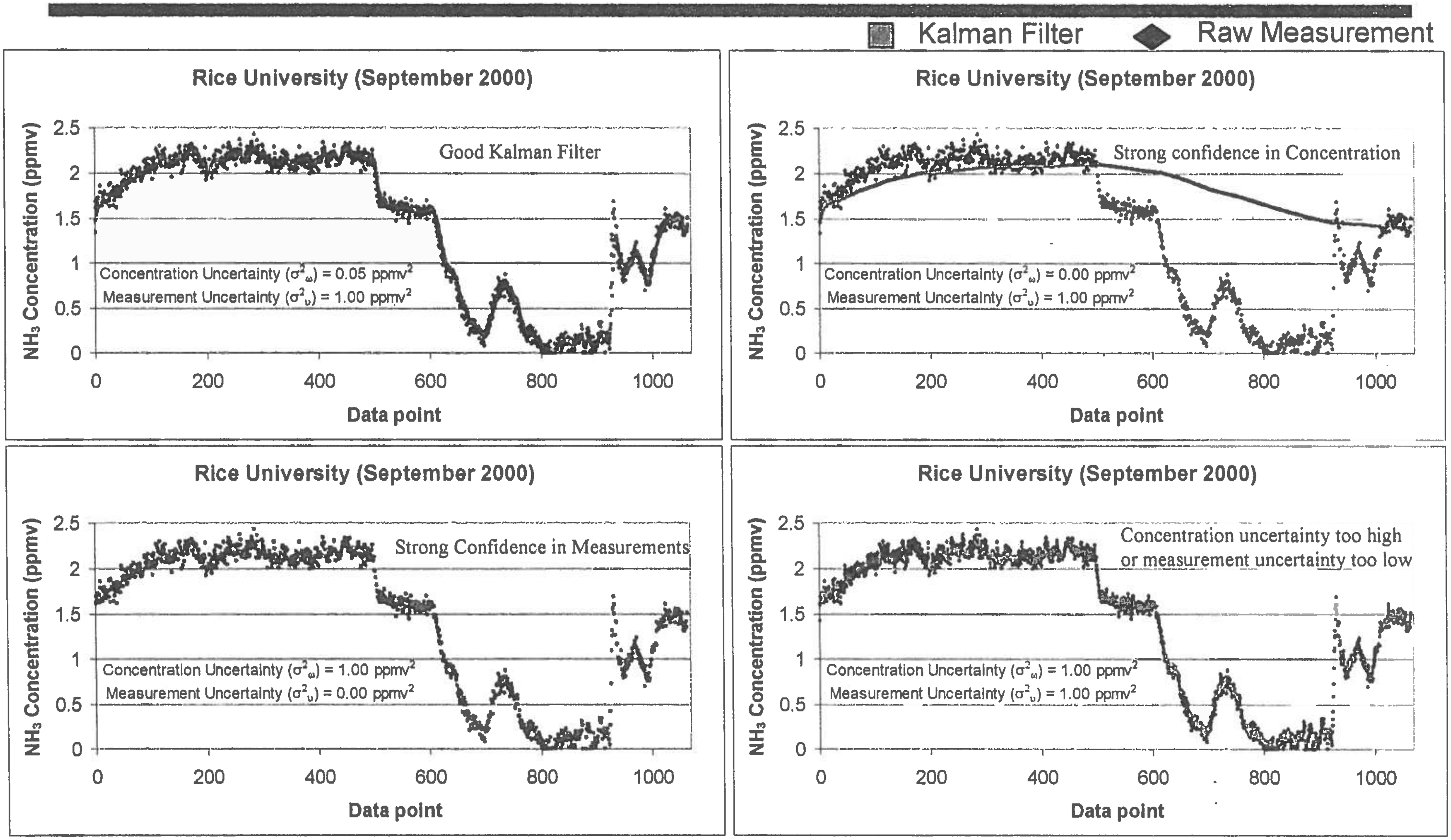
$$\lim_{\sigma_v^2 \to \infty} K_k = 0 \qquad \hat{x}_k = \hat{x}_k^-$$

New measurement "trusted" less than prediction

$$\lim_{\sigma_v^2 \to 0} K_k = 1 \qquad \hat{x}_k = z_k$$

Prediction "trusted" less than new measurement

Data Confidence



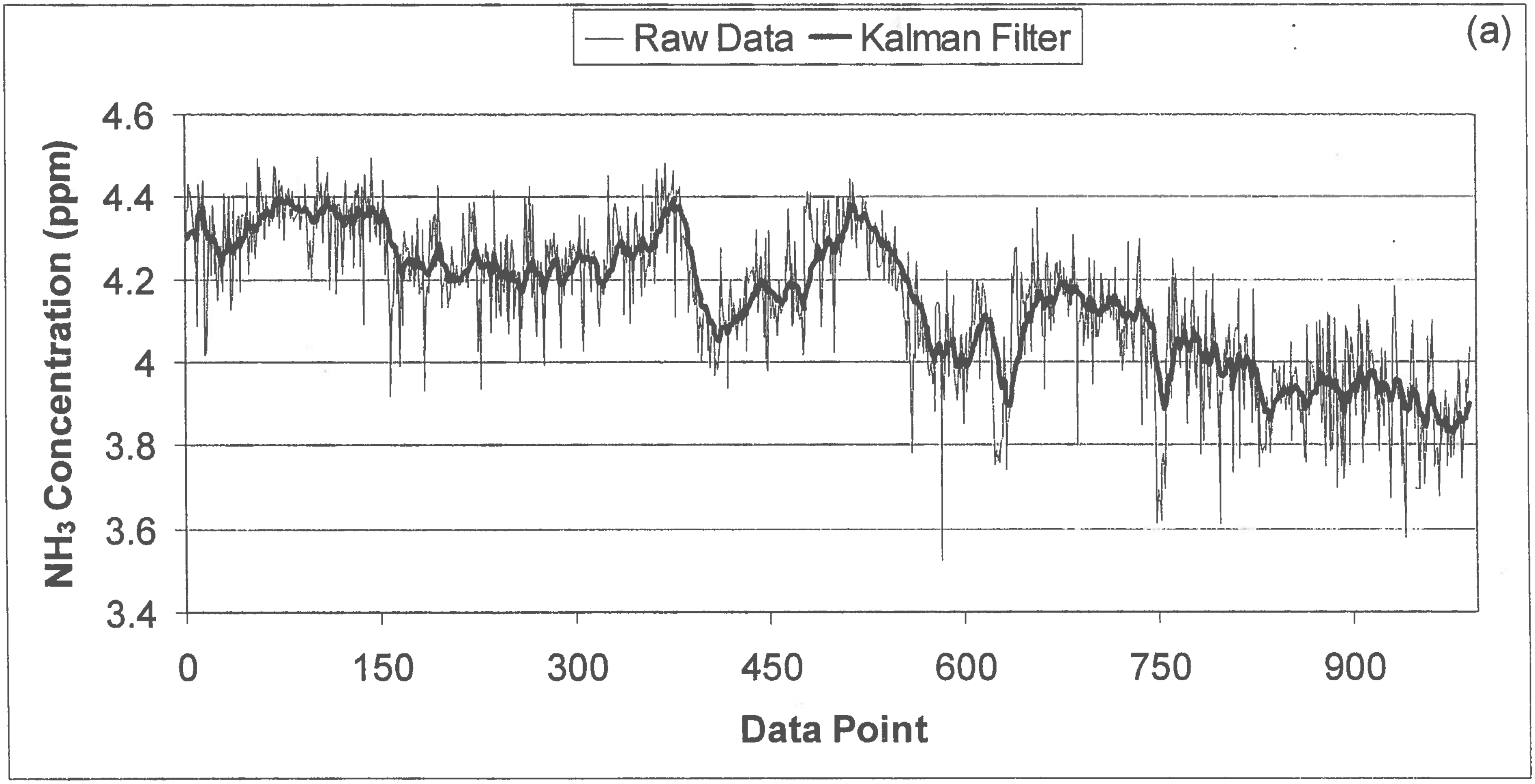
Kalman Filter Equations

$$\frac{\sigma_v^2}{\sigma_w^2} = \rho$$

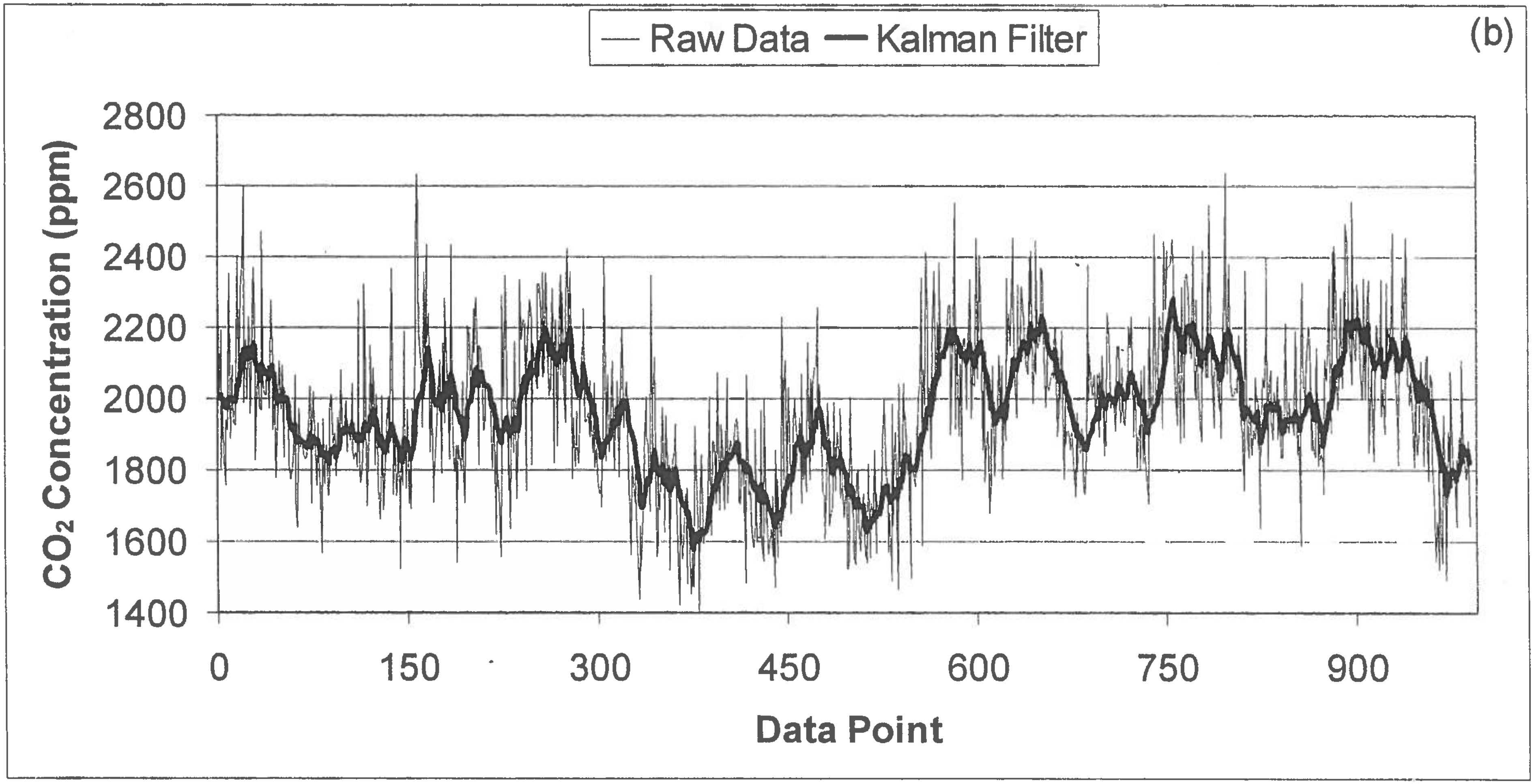
Table 3: SNR values for different values of ρ

ρ	NH_3	CO_2
50	87.0	8.2
100	23.5	8.9
150	42.0	10.1
200	49.5	10.9
250	35.0	12.2

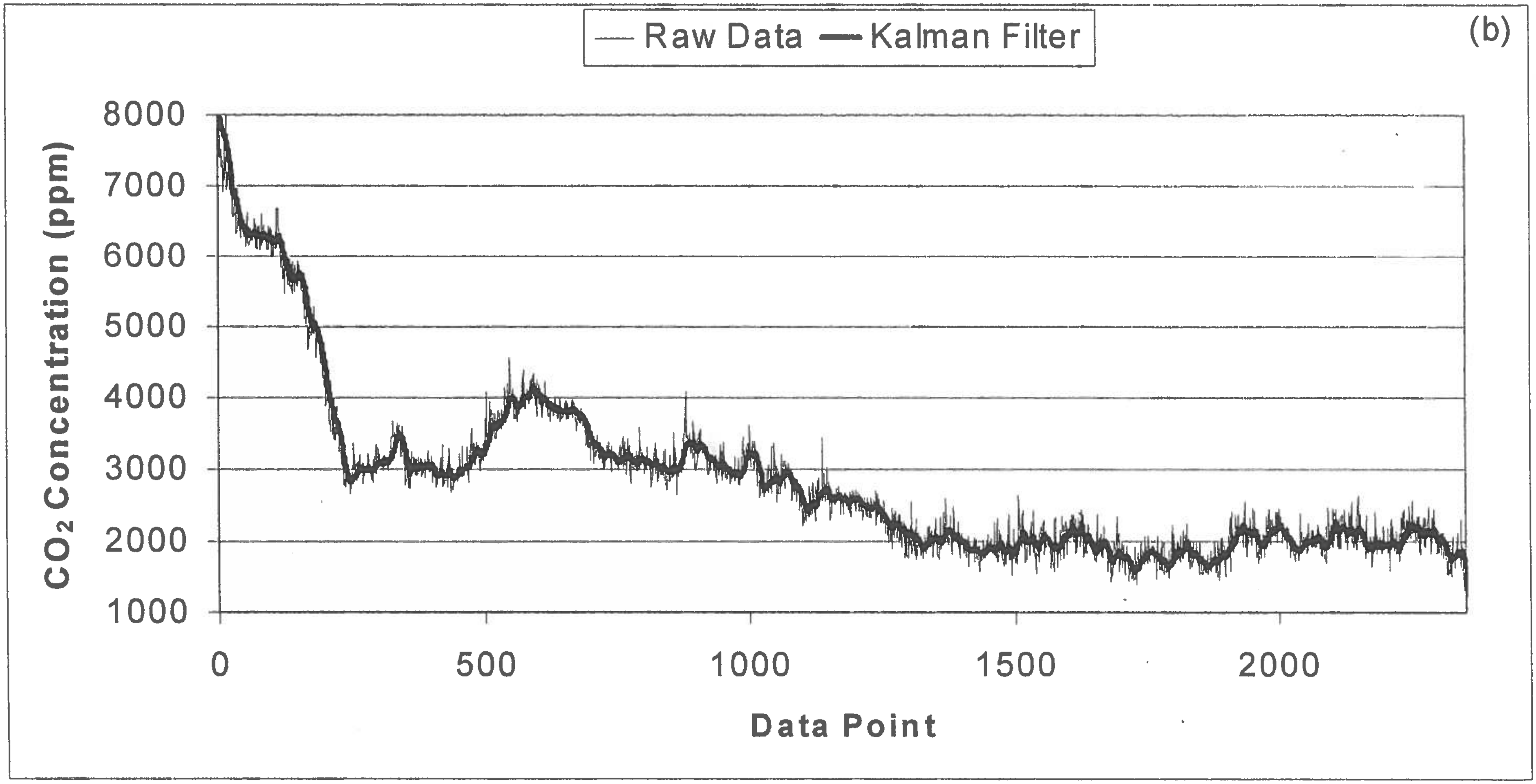
Ammonia Concentrations



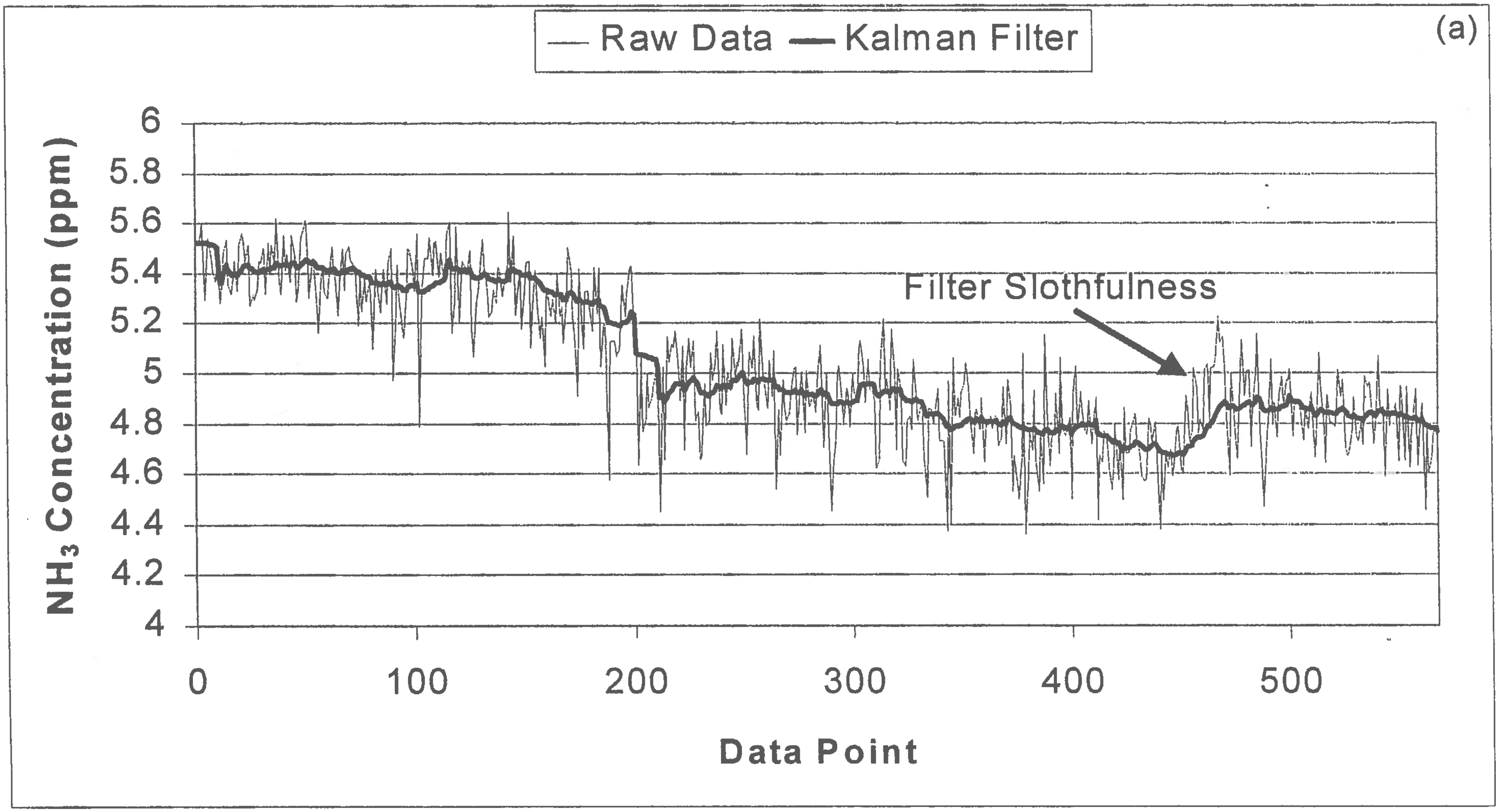
Carbon Dioxide Concentrations



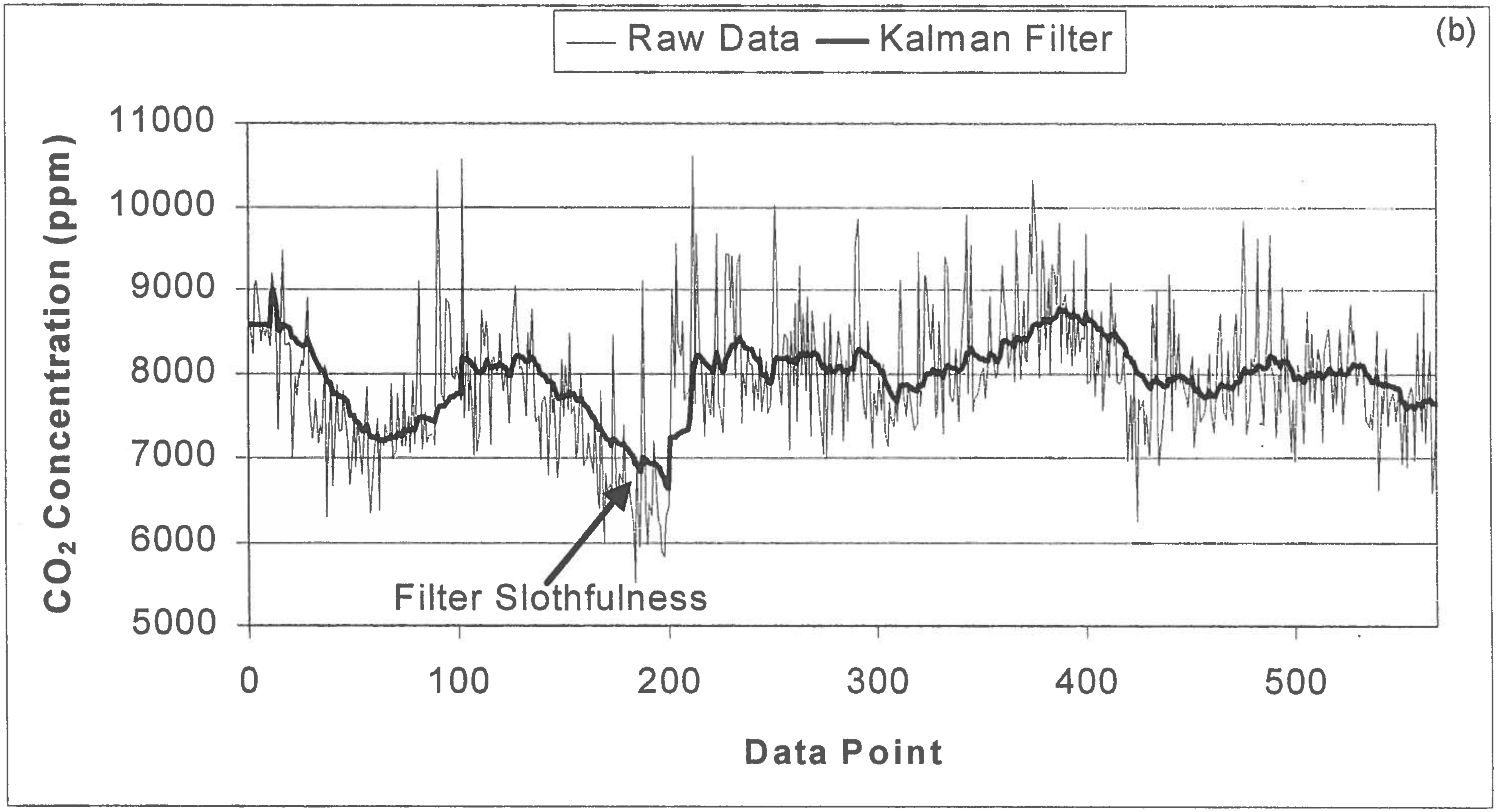
Large Dynamic Range CO_2 Concentrations



Filter Slothfulness



Filter Slothfulness



Kalman Filter Application