Superfluorescence Limitations on the Inversion Process in Large-Aperture Laser Amplifiers

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A Monte Carlo computation procedure has been used to study the effect of superfluorescence losses upon the design of a large-aperture disk laser system in terms of the enhanced photon flux density. This parameter is indicative of the maximum attainable population inversion. It was found that laser disks up to 6 in. in diam are capable of useful optical energy storage. The technique can also be applied to other laser geometries to estimate superfluorescence effects more readily than previous numerical integration methods.

I. INTRODUCTION

A single or segmented disk laser system[1] can offer a number of important advantages over the standard rod geometry. For example, high optical energy storage capability without exceeding the self-damage threshold of the laser medium, superior beam collimation and minimum phase front variations can be achieved with laser media which can be fabricated into disk form (e.g., ruby and neodymium laser glass). The merits of such a disk geometry have to be evaluated from the standpoint of achieving maximum population inversion and hence establishing the energy stored. This depends on a number of parameters such as pump intensity, duration, spectral distribution, coupling efficiency, and laser material characteristics. However, the principal factor limiting the degree of population inversion is the depletion of metastable laser atoms due to both spontaneous and amplified spontaneous emission (i.e., superfluorescence[2] or photon avalanche[3]). The presence of stimulated fluorescence leads to a nonlinear rate equation for the actual population in the metastable state. Superfluorescence occurs either by once through or total internally reflected modes and is therefore both size and gain dependent. It is the purpose of this work to discuss the limitations due to superflourescence losses on the design of large-aperture laser geometries, using the "Monte Carlo" technique (see appendix). The usefulness of the approach as compared to previous analyses[2,3] is demonstrated in the evaluation of super-fluorescence depletion losses in a laser disk configuration. The Monte Carlo method is especially useful to include losses due to internally reflected light. If desirable, the procedure can be applied equally successfully to study the performance of other novel laser geometries (e.g., spherical end surfaces instead of plane surfaces). The validity of the computation per-formed by the Monte Carlo method can be checked both by applying it to the specific rod geometry described in Ref. 2 or by comparison to an estimate of disk configuration using numerical integration methods.

II. OUTLINE OF PROCEDURE

The geometry of either a travelling wave or resonant disk laser amplifier is shown in Fig. 1. The problem of calculating superfluorescence losses in terms of the reduced density of excited atoms reduces to a con-
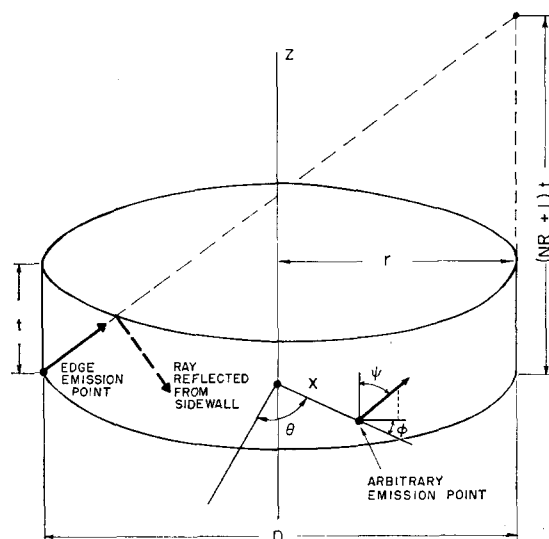

FIG. 1. Ray geometry of spontaneously emitted photons.

venient evaluation of the degree of enhanced photon flux density (Φ) stimulated into existence from a single spontaneous decay by all differential volume elements of the disk configuration. The following expression for Φ includes all possible emission points inside and at the edge of the disk for all possible emission directions. Since the thickness of the laser disk is essentially fixed by effective and uniform pumpability, mechanical rigidity, and minimum thermal distortions, the maximum allowable disk cross section for optimum optical gain becomes the critical design parameter.

[1] R. E. Kidder, J. E. Swann, K. Pettipiece, F. Ramer, E. D. Baird, and B. Loth, Int. Quantum Electron. Conf., Miami, Fla., 1968.
[2] L. Tonks, J. Appl. Phys. **35**, 1134 (1964).
[3] P. V. Avizonis and W. R. Willoughby, J. Appl. Phys. **37**, 682 (1966).

Now,

$\Phi =$ (Rate of spontaneous decay per unit volume) (amplification) (angular distribution) (volume)

$$= \frac{N}{4\pi} \int_{x=0}^{r} \int_{\theta=0}^{2\pi} \int_{z=0}^{t} \int_{\phi=0}^{2\pi} \int_{\Psi=0}^{\pi} \exp[\alpha l(x, z, \phi, \Psi)] \sin\Psi d\Psi d\phi dx dz x d\theta$$

or

$$\frac{\Phi}{N} = \int_{x=0}^{r} 2x \int_{x=0}^{t/2} \int_{\phi=0}^{+1} \int_{\Psi=0}^{\pi} \exp[\alpha l(x, z, \phi, \psi)] \sin\Psi d\Psi d\phi dz dx$$

$$= \int_{x=0}^{r} 2x \int_{z=0}^{t/2} \int_{w=-1}^{+1} \int_{\phi=0} \exp[al(x, z, w, \phi)] d\phi, dz, dx, dw,$$

where $w = -\cos\Psi$. N is the number of spontaneous emitted photons per second per unit volume within the disk [equivalent to (n/τ), if n is the number of metastable atoms and τ is the spontaneous lifetime]

x, z, θ are the coordinates of the photon source,

Ψ, ϕ specify the polar and azimuthal directions respectively,

r is disk radius,

t is disk thickness,

l is the effective optical path length in the laser medium, i.e., $l = f(x, z, \phi, \Psi)$ independent of θ for a circular geometry,

α is the amplification coefficient ($\alpha = \sigma n$, if σ is the stimulated absorption cross section).

To simplify the calculation certain approximations were applied. These include:

(1) Negligible spatial and time-dependent variations of population inversion in the medium.

(2) Negligible polarization effects.

(3) No cross relaxation and laser linewidth narrowing. The latter can be taken into account following the procedure described in Ref. 2.

(4) No reflection effects at either the cylindrical air–glass interfaces (since these can be minimzed by design) or for angles less than the critical angle (θ_c) at the end faces.

(5) No scattering or absorption losses.

III. DISCUSSION

The total rate of depumping was calculated for two thicknesses (0.5 and 1 in.) and several diameters ranging from 3 to 6 in. for both single photon avalanche transits and multiple reflections in a neodymium laser glass disk with $\mu = 1.58$ and $\alpha = 0.3$ cm⁻¹. Photon origin, size, and refractive index (μ) of our optically charged material determine the maximum number of possible reflections NR_{max} within the disk. From Fig. 1 it is seen that

$$\tan\theta_c \geq D/(NR_{max}+1)t,$$

where $\theta_c = \sin^{-1} 1/\mu$.

The results of a Monte Carlo computation of the above integral expression are shown both in tabular (Table I) and graphical form (Fig. 2). Hence the value of Φ can be used to find the reduced forward gain $G = \exp(\sigma N_{eff} l)$ by calculating the effective metastable state population (N_{eff}) and comparing it with the case in which no significant superfluorescence occurs. If a practical (but arbitrary) limiting ratio of Φ/N of 100 is chosen, then this implies from Figs. 2 and 3 that a disk of maximum diam of 6 in. and effective thickness 0.5 in. is practical as a large aperture disk laser amplifier before the superfluorescence loss becomes sufficiently serious to degrade performance.

For a ruby rod of length 17.78 cm, 0.952 cm in diam, with $\mu = 1.77$ and $\alpha = 0.3$ cm⁻¹ the Monte Carlo pro-

TABLE I. Φ/N values for both single and multiple photon trajectories in Nd glass disk.[a]

t	r	No. reflections	$\langle \exp(\alpha l) \rangle$	Φ/N
1.0	1.5	0	2.09	14.8
0.5	1.5	0	1.67	5.9
1.0	1.5	3	3.35	23.7
0.5	1.5	7	3.29	11.6
1.0	2.0	0	2.49	31.3
0.5	2.0	0	1.89	11.9
1.0	2.0	4	5.71	71.8
0.5	2.0	12	5.66	35.6
1.0	2.5	0	3.02	59.3
0.5	2.5	0	2.16	21.2
1.0	2.5	6	10.4	204.0
0.5	2.5	12	10.4	102.0
1.0	3.0	0	3.79	107.0
0.5	3.0	0	2.54	35.8
1.0	3.0	7	20.3	574.0
0.5	3.0	14	20.3	288.0

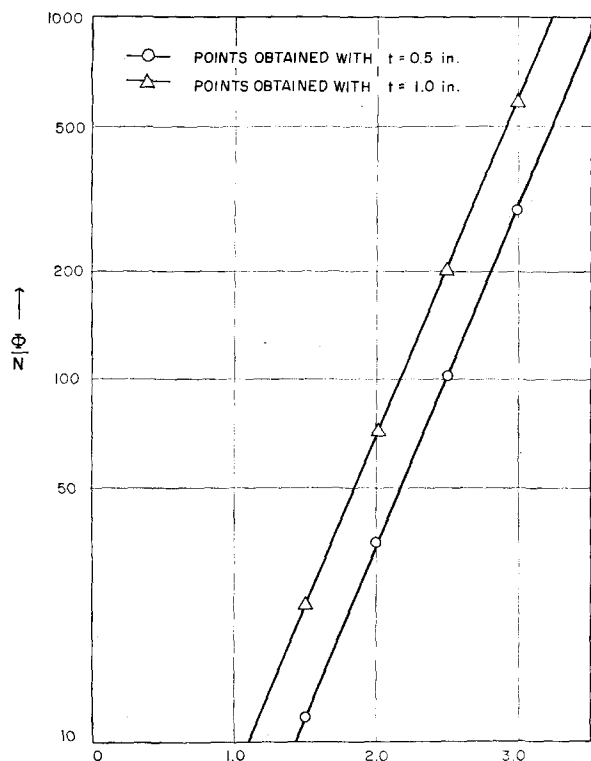[a] $\alpha = 0.762$ in.⁻¹ (0.3 cm⁻¹) and $\theta_c = 39.26°$.

FIG. 2. Total internal reflection values of Φ/N as a function of the Nd glass disk radius for $t=0.5$ in. and 1.0 in.

cedure yields a value of 81.44 for Φ/N neglecting any internal reflections. This result indicates that for such a rod and gain factor, one should expect only modest superfluorescence effects, which is in agreement with the previous analysis by Tonks.[2]

ACKNOWLEDGMENTS

APPENDIX: THE MONTE CARLO METHOD

From Sec. II we have that

$$\frac{\Phi}{N} = r \int_{x=0}^{r} \frac{2x}{r} \int_{z=0}^{t/2} \int_{w=-1}^{1} \int_{\phi=0}^{\pi}$$

$$\times \exp[\alpha l(x, z, w, \phi)] d\phi \, dw \, dz \, dx.$$

If we generate points (x, z, w, ϕ) at random so that x has probability density $2x/r$ on $[0, r]$ and z, w, ϕ are uniformly distributed on the appropriate intervals, then the average value $\hat{e}$ of $\exp[\alpha l(x, z, w, \phi)]$ satisfies

$$(\Phi/N) \approx \pi r^2 t \hat{e}$$

with an error which tends to zero (with probability one) as the number of points used tends to infinity. The variance of the samples $\exp(\alpha l)$ can be used to obtain an estimate of the error in $\hat{e}$ at any stage in the calculation. (See Ref. 4 for a general discussion of quadrature by Monte Carlo methods.) The random numbers required were obtained with a standard multiplicative congruential generator, using 48-bit arithmetic.

The principal advantage of the Monte Carlo approach here is that low-accuracy results (which are entirely adequate) can be obtained with only a modest number of evaluations of the integrand. Moreover, the value $\hat{e}$ is a constantly improving approximation to the desired result, so the calculation can be terminated immediately when a sufficient number of trials have been made. The results given in Table I should be in error by at most 4% for values of $\Phi/N < 150$, 7% for larger values.

The quantity $l(x, z, w, \phi)$ was calculated using a rather general ray-tracing subroutine. For the simple case of the right-circular cylinder a more direct approach would have been more efficient, but the ray-tracing program can be used with much more complicated configurations.

[4] H. Kahn, "Multiple Quadrature by Monte Carlo Methods," *Mathematical Methods for Digital Computers*, A. Ralston and H. Wilf, Eds. (John Wiley & Sons, Inc., New York, 1960).