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Parametric Photon Interactions and Their Applications

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Abstract

The recent achievement of obtaining second and third harmonic radiation at optical frequencies with an intense monochromatic laser beam[1,2,3] has led to great interest in phenomena resulting from nonlinear interactions between radiation and matter.

The mechanisms of these harmonic generations can be identified as parametric interactions. In particular, the concept of traveling-wave parametric interaction can be applied to enhance these interactions, as was demonstrated by Giordmaine,[2] and Maker, et al.[3] Furthermore, these interactions can be interpreted as typical examples of three-dimensional parametric interactions of photons as quasi-particles.[4]

The purpose of this paper is to analyze optical frequency parametric interactions. The basic mechanism and selection rules of these parametric interactions will be described, not only for harmonic generation, but also for parametric amplification and frequency conversion processes. The experimental approach to achieve these interactions and some of our preliminary experimental results will be presented. The application of these parametric interactions to potential millimeter wave and infrared devices will be discussed.

Introduction

The recent achievement of generating harmonic radiation at [illegible] frequencies[1-3] and observing mixing of light beams[5] has led to great [illegible] in the general area of nonlinear optical phenomena. These nonlinear [illegible] effects can be interpreted as typical examples of parametric interactions.[7,8] The purpose of this paper will be to discuss the basic mechanism of optical parametric interactions. This analysis can be used to establish the necessary requisites for the design of traveling-wave parametric devices.

* [illegible]

** [illegible]

The Basic Concept of Parametric Interactions

At radio and microwave frequencies, electrical resonances can be excited in lumped-circuit or cavity resonators. When three resonators are coupled together through a nonlinear reactance, frequency mixing occurs. If the sum or difference between two of the resonant frequencies coincides with the third resonant frequency, there can be an exchange of energy among these three resonators. Then, energy can be pumped by one resonator to excite the other two resonators to achieve parametric amplification or frequency conversion. For the degenerate case of two harmonically related resonators, parametric harmonic generation or subharmonic amplification can be obtained.

At radio frequencies, the dimension of a lumped-circuit resonance is much smaller than the wavelength. At microwave frequencies, the dimension of a cavity resonator becomes comparable to the wavelenght λ. Thus, the parametric interaction using resonators is confined to a space not larger than λ^3. When we extend this concept to optical frequencies, the interactions can be regarded as point interactions. However, it is possible to expand the interaction volume for much enhanced parametric interactions by means of the traveling-wave effect as was demonstrated by Giordmaine[2] and Maker[3] for optical harmonic generation. The basic concept of traveling-wave parametric interactions can be explained as follows:

A traveling wave with frequency ω and propagation constant $\vec{\beta}$ can be represented by the wave equation,

$$\nabla^2 E - \frac{\partial^2}{\partial t^2}(\mu \varepsilon E) = 0 \qquad (1)$$

where E is the field intensity of the wave, for example, the electric field of an electromagnetic wave. The values of the permeability, μ, and permitivity, ε, determine the velocity of propagation. Equation 1 can also be represented in complex form as

$$\nabla^2(E + E^*) - \frac{\partial^2}{\partial t^2}[\mu \varepsilon (E + E^*)] = 0. \qquad (2)$$

If one considers the generalized case of many traveling waves being propagated in the same medium, Equation 2 can be rewritten as

$$\nabla^2 \sum (E + E^*) - \frac{\partial^2}{\partial t^2}\left[\sum \mu \varepsilon (E + E^*)\right] = 0. \qquad (3)$$

When the propagating medium is nonlinear, intense pump radiation, such as that obtainable from a laser beam, would create a traveling wave disturbance in the velocity of propagation. This effect can be expressed in terms of the perturbed wave equation

$$\nabla^2 \sum (E + E^*) - \frac{\partial^2}{\partial t^2} \left\{ \mu_0 \varepsilon_0 \left[1 + \xi(\omega_p, \vec{\beta}_p) + \dots \right] \sum (E + E^*) \right\} = 0 \qquad (4)$$

where $\xi(\omega_p, \beta_p)$ is the pump factor determined by the nonlinearity of the medium.

Let us select two individual traveling waves, say E_s for a signal wave and E_i for an idler wave. If ω_s, $\vec{\beta}_s$ and ω_i, $\vec{\beta}_i$ are, respectively, the frequency and propagation constants of the signal and idler, and

$$\omega_p = \omega_i + \omega_s, \qquad \vec{\beta}_p = \vec{\beta}_i + \vec{\beta}_s. \qquad (5)$$

Then, the signal and idler traveling waves can be coupled together through the pump perturbation as

$$\nabla^2 E_s - \mu_0 \varepsilon_0 \ddot{E}_s = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \left\{ \xi(\omega_p, \vec{\beta}_p) E_i^*(-\omega_i, -\vec{\beta}_i) \right\}$$
$$\nabla^2 E_i - \mu_0 \varepsilon_0 \ddot{E}_i = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \left\{ \xi(\omega_p, \vec{\beta}_p) E_s^*(-\omega_s, -\vec{\beta}_s) \right\} \qquad (6)$$

Equation 6 shows that the signal and idler traveling waves, corresponding to the left-hand side of the equation, can be excited and amplified by the parametric perturbations of the pump, as shown on the right-hand side of the equation. In a similar manner, other traveling waves in Equation 4 can be coupled together parametrically to produce frequency conversion, harmonic or subharmonic generation, or interactions involving more than three frequencies. Equations 5 and 6 can be rearranged to show these interactions.

Equation 5 gives the selection rules of the coupling of traveling waves for parametric amplification. These relations are analogous to the conservation laws of energy and momentum. The vector relationship for the propagation constants indicates that the parametric interaction can be extended to three-dimensional space.[4] The actual interaction volume is the portion of the medium which is passed through by all three waves. In practice, when a laser beam of small cross-sectional area is used as the pump, the arrangement for the largest interaction volume is usually to keep all the waves in the same path; i.e., the vector relationship in Equation 5 is reduced to the one-dimensional equation developed by Tien.[5]

The Mechanism of Forward and Backward Traveling Wave Interactions

The performance of the parametric interactions of coupled traveling waves can be analyzed by solving Equation 6. In general, the parametric excitation of the pump introduces a perturbation in the propagation constants of the other traveling waves. Let the perturbed propagation constants of the signal and idler waves be $\vec{\beta}'_s$ and $\vec{\beta}'_i$, respectively. Then, remembering the selection rules of Equation 5, we may put the perturbation of the propagation constants $\vec{\Delta\beta}$ as

$$\vec{\beta}'_s = \vec{\beta}_s + \vec{\Delta\beta}, \qquad \vec{\beta}'_i = \vec{\beta}_i - \vec{\Delta\beta} \qquad (7)$$

From Equations 6 and 7, the value of $\vec{\Delta\beta}$ can be solved as

$$\vec{\Delta\beta} = |\Delta\beta|\, \hat{\gamma} = \pm \frac{i}{2} \cdot \frac{\xi\, |\vec{\beta}_i|\, |\vec{\beta}_s|}{\sqrt{(\vec{\beta}_i \cdot \hat{\gamma})(\vec{\beta}_s \cdot \hat{\gamma})}} \cdot \hat{\gamma} \qquad (8)$$

where $\hat{\gamma}$ denotes the unit vector for the perturbation of the propagation constant. Since the perturbation is caused by the pump, the direction of $\hat{\gamma}$ is normally along the direction of the pump traveling wave or very close to it.

According to Equation 8, the characteristics of the parametric excitations can be classified as either forward- or backward-traveling wave interactions, depending upon the relative directions of the three traveling waves. In the case of forward interactions, all three waves travel along the same general direction, and the denominator in Equation 8 becomes positive. Thus, the perturbed propagation constants become complex quantities, and the waves are amplified with exponential gain. The amplification increases with the nonlinearity of the medium and the interaction length. The backward interaction corresponds to the case when one of the traveling waves, say the signal wave, travels nearly along the opposite direction. Then, the denominator in Equation 8 becomes imaginary and the perturbation $\Delta\beta$ is a real quantity. The effect of the perturbation produces an inherent regeneration among these traveling waves. This operation has high gain and may become unstable; thus, the forward interaction is primarily useful for amplification and frequency conversion, while the backward interaction lends itself to the design of both amplifiers and oscillators with extremely wide tuning ranges. In view of the conditions set by Equation 5, it is impossible to achieve parametric interaction between a forward pump traveling wave and backward signal and idler traveling waves. Thus, the forward and backward interactions discussed above are the two general types of interactions.

Selection Rules

The selection rules of Equation 5 can be combined to give the information on the frequency ratio as a function of the velocity ratio of the three traveling waves. For optical frequencies, it is more convenient to use the indices of refraction n_p, n_i, and n_s for the pump, idler, and signal waves respectively. Figure 1 shows the relationship of the selection rules for the

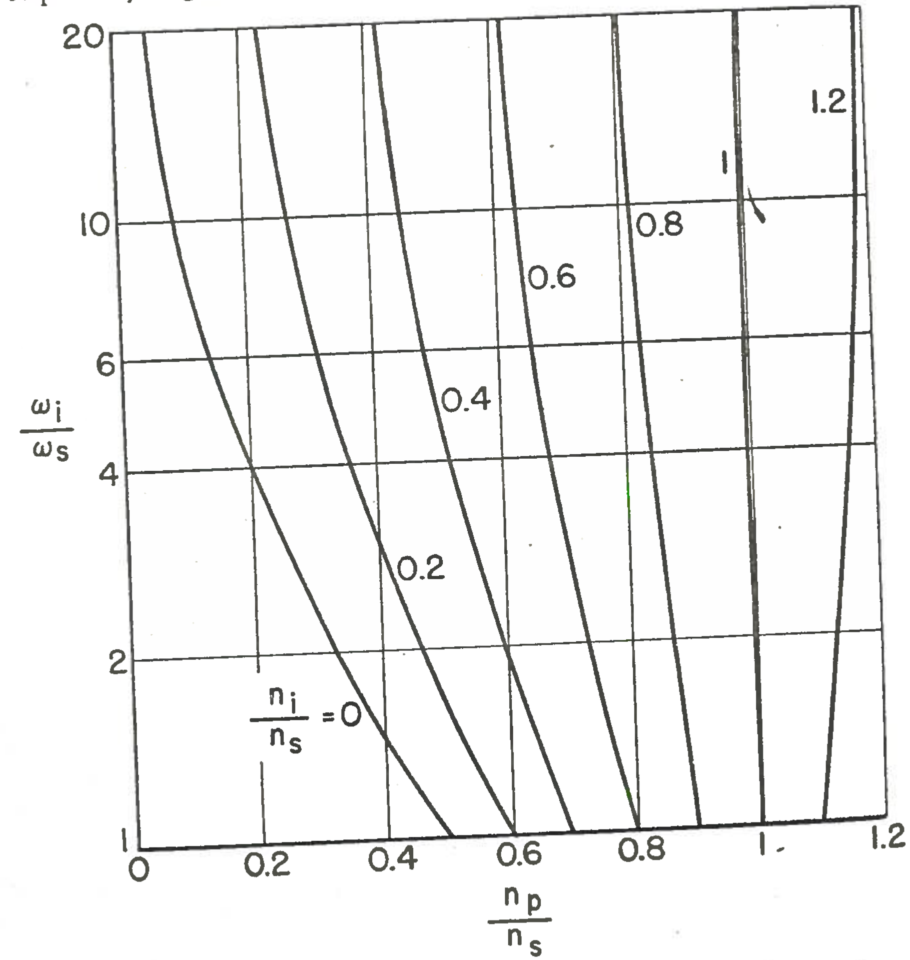

Fig. 1—Frequency ratio *vs.* relative indices of refraction in forward traveling-wave interactions.

forward traveling wave interactions in terms of the ratio of the various indices of refraction and corresponding frequency ratios. The condition that all three indices of refraction are equal describes the well-known condition for index matching applied to optical harmonic generation.[1,2]

Figure 1 shows that it is possible to achieve parametric interactions with other combinations of the frequency ratio and relative indices of re-

fraction. For example, it is possible to combine an ordinary ray and an extraordinary ray in a crystal to generate the optical second harmonics. In the particular case of potassium dihydrogen phosphate (KDP), the material

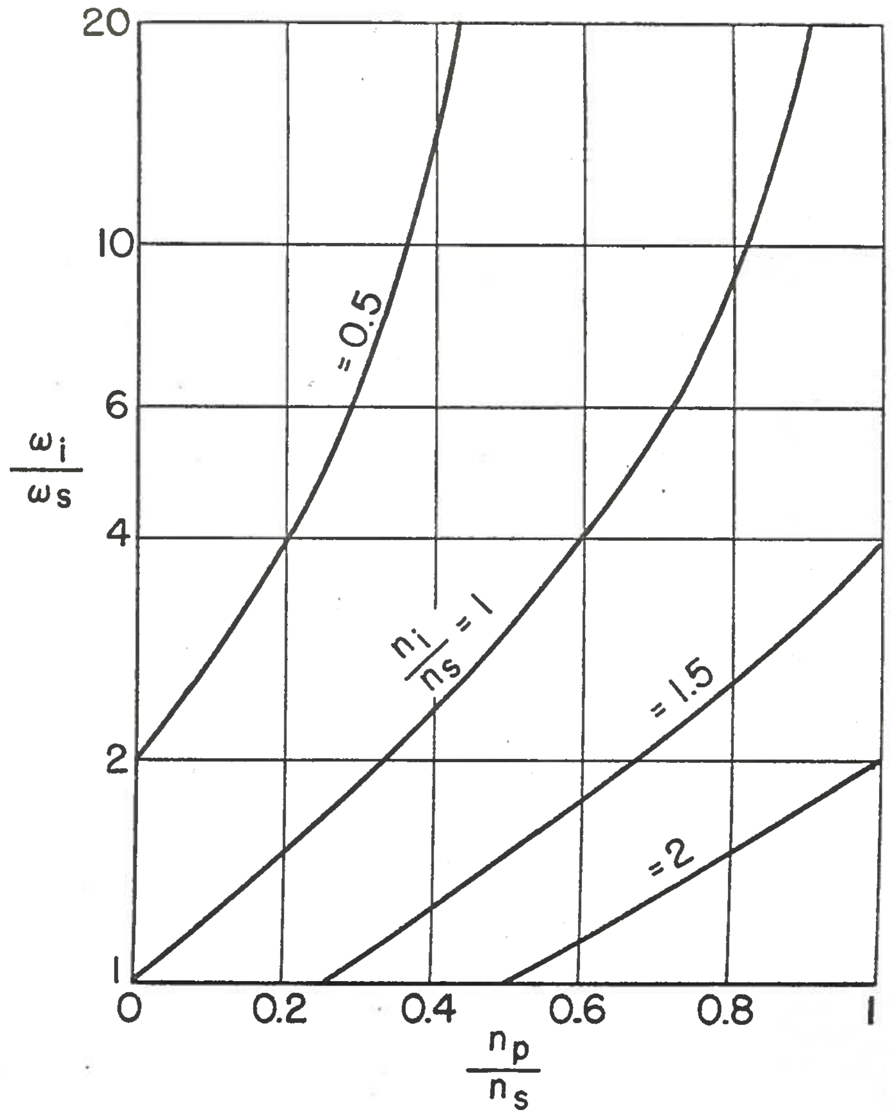

Fig. 2—Frequency ratio *vs.* relative indices of refraction in backward traveling-wave interactions.

used most frequently in the generation of optical second harmonic radiation, it is possible to achieve parametric interaction with a frequency ratio (ω_i/ω_s) of 1 [illegible] between an ordinary ray and an extraordinary ray along a direction at 90° to the optical axis of the crystal. The limitation in the forward interactions is that the three indices must either be equal to one another, or be all different. Thus most crystals can only be used within a limited range of frequencies when the signal and idler frequencies are very close together.

Figure 2 shows the characteristics of backward traveling wave interactions. In this case, the frequency ratio is usually very high, even when the indices of refraction are very close to one another, as is usually the case

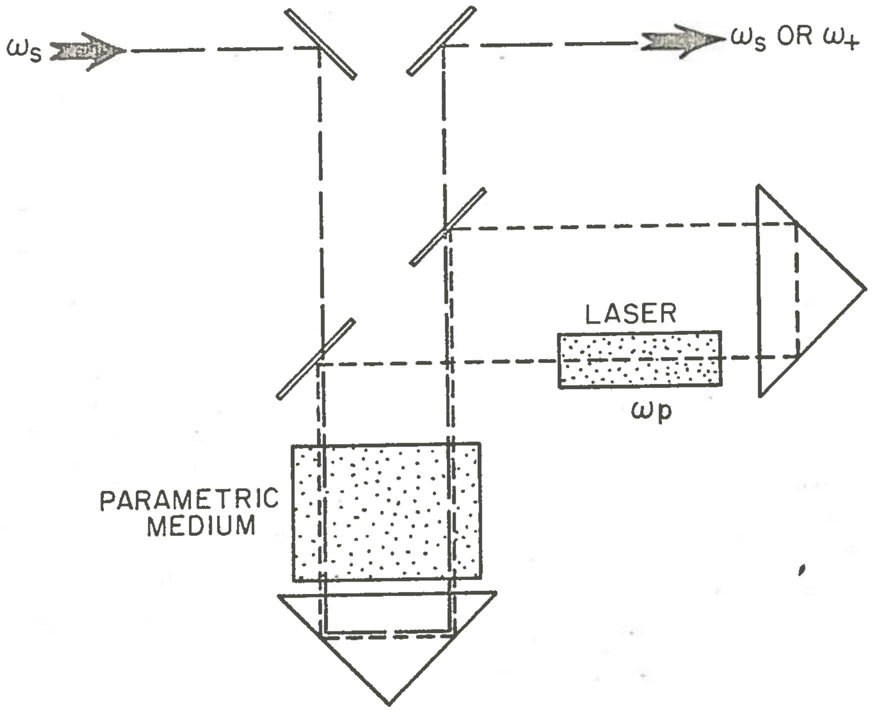

Fig. 3—Proposed experimental arrangement.

for optical media. Thus, backward wave interactions may be extremely useful for accomplishing the design of continuously variable optical oscillators with considerably wide tuning ranges.

Applications

To investigate the applicability of parametric interactions to the development of new optically pumped devices, the particular experimental arrangement in Figure 3 is proposed. The setup consists of a resonant structure of two roof prisms or corner reflectors which possess the properties of an optical traveling wave cavity.[10] Between the two prisms are located the laser, as the parametric pump source, and the nonlinear parametric medium

Depending on the optical pump power requirements, the laser can b operated either under normal conditions or in a manner which provides 'giant' pulse laser output using specially developed optical Q-switching tech niques.[11] The use of special mirrors with selective reflectivity should insur the necessary forward or backward traveling wave coupling at the sign and idler frequencies. This configuration could then be used for studyin experimentally the feasibility of amplification and generation of coherer optical radiation or frequency conversion to the upper side band frequenc ω_+.

The following design parameters are most important in the plannin of a successful experimental scheme:

(1) The careful choice of a suitable nonlinear material, capable of su porting the various interaction effects with minimum losses, essential.

(2) Maximum conversion efficiency is desirable. This depends not on on the nonlinear medium, but also on the existing optical pow level. Ultimately the conversion efficiency is limited by the wel known Manley-Rowe relationship.

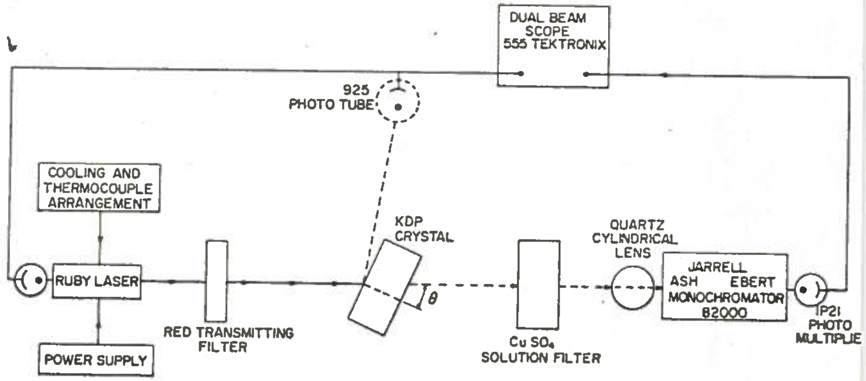

Fig. 4—Schematic of experimental arrangement.

(3) Effective index matching for maximum power transfer is importan The importance of proper orientation for the nonlinear mediun has already been stressed. A convenient experimental arrangement shown in Figure 4, has been assembled to assist in the interpretation of the selection rules and correct phase matching conditions for numerous potential nonlinear materials. Typical observations obtained in this manner using KDP are shown in Figure 5. The setup allows a slow rotation of the nonlinear medium, normal with respect to the incident laser beam. The traces with the expanded time scale give an indication of the correlation involved in generating second harmonic radiation. There is some variation in the

sweep rates for the two beams of the scope. This accounts for the slight discrepancies in coincidence of laser spikes in the end portions of the oscillograms.

Finally, it should be pointed out that there is a great potential in the development of new optically pumped parametric devices— not only as amplifiers or oscillators, but also as detectors, modulators, power limiters[11]—

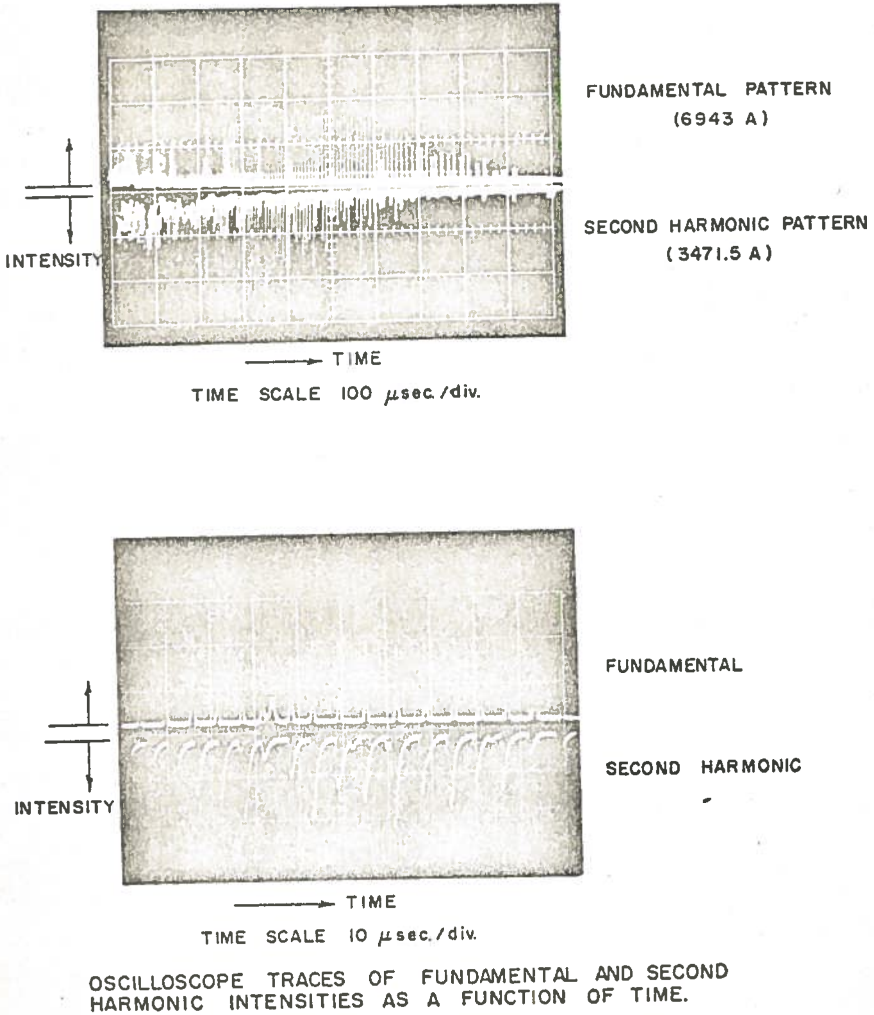

Fig. 5—Oscilloscope traces of fundamental and second harmonic intensities as a function of time

and the study of various coherent nonlinear scattering processes such as the Raman effect. By means of parametric photon interaction methods, it will be feasible to overcome the existing lack of convenient experimental techniques in the region of the electromagnetic spectrum between millimeter wave and infrared frequencies.

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Electro-Optic Light Modulators

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Abstract

Some recent work connected with microwave modulation of light by the electro-optic effect is reviewed.

The efficiency of the cavity-type modulator[1] has been improved by the use of a longer thinner KDP rod. This device has been employed to provide a continuous modulation of the visible helium-neon gas maser at 9 Gc. The microwave sidebands on the light are observed directly as a splitting of the rings of a Fabry-Perot etalon.[2]

The dielectric constant and loss tangent have been measured in KDP as a function of temperature in order to determine the feasibility of operating KDP modulators at reduced temperature.[3]

Proposals and calculations related to wide-band travelling-wave light modulation have been made. The characteristics of coaxial or parallel plate transmission lines, partially filled with KDP, are calculated[4] and two modulation schemes are considered: the light travels along the propagation direction[4] in the transmission line, or the light follows a zig-zag path about this direction.[5]

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