The magnetic moment of the proton
II. The value in Bohr magnetons

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(*Communicated by B. Bleaney, F.R.S.—Received* 6 *July* 1962)

The magnetic moment of the proton has been measured in Bohr magnetons by comparing the proton spin-resonance frequency with the electron cyclotron frequency in the same magnetic field. The electron cyclotron signal was observed as an absorption of energy from a radio-frequency field in a cavity. Particular attention was paid to the elimination of effects which might have altered the electron cyclotron frequency from the true value. The result, related to the spin-resonance frequency of the free proton, is that the magnetic moment of the proton is $(1{\cdot}521043 \pm 0{\cdot}000006) \times 10^{-4}$ Bohr magneton.

1. Introduction

It is well known (see the preceding paper, Sanders & Turberfield 1963) that the proton magnetic moment in nuclear magnetons (μ_p/μ_n) is equal to the ratio of the proton spin-precession frequency ν_n to the proton cyclotron resonance frequency ν_c in the same magnetic field. By substitution of the mass of the electron m for the mass of the proton M throughout, it follows that the proton magnetic moment in Bohr magnetons (μ_p/μ_0) is equal to the ratio of the proton spin-precession frequency ν_n to the electron cyclotron frequency ν_e in the same magnetic field. The comparison of these two frequencies has been carried out in a magnetic field of about 3500 G in which ν_n and ν_e are close to 15 Mc/s and 10 Gc/s respectively. The electron cyclotron frequency was measured by detecting the resonant absorption of microwave energy by free electrons in an evacuated cavity situated in the magnetic field. Several significant corrections to the measured frequency have to be considered in order to derive a value for ν_e. The value of ν_n was measured by well-established techniques which are discussed in the preceding paper.

2. Apparatus

The magnetic field

The same magnet was used for this measurement as for the measurement of μ_p/μ_n described in the preceding paper. In order to accommodate the microwave cavity it was necessary to increase the gap width of the magnet to $1\frac{1}{2}$ in. To produce a field of about 3500 G additional low-resistance coils were used, supplied with a current of 5 A from a current-stabilized source, driven, with preliminary voltage stabilization, from the a.c. mains. These coils provided about half the total excitation, the rest coming from the high-resistance coils used previously which were fed from a supply controlled by a proton resonance head. The total value of the magnetic field was thus controlled by the proton resonance stabilizer, and improved techniques such as preliminary voltage stabilization of the current supply to the high resistance coils gave a magnetic field stability of 1 p.p.m. during the measurement of μ_p/μ_0. The same technique as before was used for varying, over a small range, the magnetic field at the cavity, that is, passing a steady current of variable magnitude through the coils used to modulate the field at the proton sample used for stabilization.

The microwave cavity

The cavity was a hollow right circular cylinder of inside dimensions 4·6 cm diameter and 2·8 cm high operated in the TE_{101} mode. The TM_{111} mode, which coincides in frequency with the TE_{101} mode, was suppressed by constructing the cavity (figure 1) in two parts electrically insulated from each other. The insulating ring has negligible effect on the Q of the TE_{101} mode, but because of the suppression of the

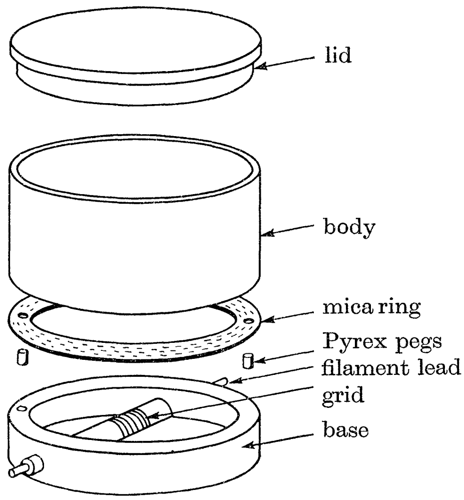

FIGURE 1. The microwave cavity.

wall currents which would flow across this boundary in the TM_{111} mode the latter resonance was reduced to negligible proportions. In addition, the method of coupling to the cavity favoured the TE_{101} mode.

The cavity was made from tellurium copper and was silver plated internally and externally. Tellurium copper was found to be a material which showed negligible magnetic susceptibility, and was better in this respect than samples of reputedly pure copper. The walls of the cavity were 0·25 mm thick, except where for constructional reasons an increase was desirable, and the measured unloaded Q of the cavity was 6000.

Diametrically across the inside of the lower end of the cavity was a semi-cylindrical ridge which in the central region was formed from a grid of molybdenum wires 0·05 mm in diameter spaced by 0·25 mm. Inside the ridge were tungsten rods carrying at their centre a tungsten filament 0·1 mm in diameter to act as a source of the electrons which entered the cavity through the grid. Microwave input and output coupling was made to the cavity by 0·7 mm diameter copper rods which entered the cavity through 2·5 mm diameter holes at diametrically opposite points halfway up the cavity walls. The inner ends of these copper rods were bent at right angles in a plane perpendicular to the cavity axis to couple to the TE_{101} mode of the cavity.

The cavity was enclosed in a cylindrical glass envelope provided with tungsten seals connected to the two microwave coupling rods and to the two ends of the filament. Two additional tungsten seals provided electrical connexions to the two insulated parts of the cavity walls. The glass envelope was mechanically mounted so that it could be removed from the magnet and subsequently replaced in its former position with an uncertainty of ± 1 mm. The value of the magnetic field strength at any point in the cavity during a determination could thus be found with a small uncertainty by plotting the field distribution in the absence of the cavity before and after each determination. The top of the glass envelope was provided with an indentation to accommodate a small liquid-paraffin proton resonance sample used as a reference during the determination of ν_e. A coil 7 cm in diameter wound with 400 turns of 32 s.w.g. wire supplied with 70 c/s current provided a small modulation of the magnetic field both at the proton resonance sample and at the electrons which provided the cyclotron resonance signal. All parts of the cavity and its surroundings were carefully tested, using a proton resonance head, to ensure that they did not disturb the magnetic field intensity at any point by more than ± 1 p.p.m.

The vacuum system

Preliminary experiments showed that traces of organic contamination in the cavity led to insulating films on the cavity walls which became charged and made the distribution of potential inside the cavity uncertain. For this reason the cavity and its envelope were designed so that the whole could be baked at a temperature of 400 °C, which was successful in removing the insulating films. The system was evacuated with a fractionating oil pump provided with a liquid-nitrogen trap. This trap was kept cold during the whole period of the determinations, and, with the exception of the mica insulating ring in the cavity, only metal and glass were used on the cavity side of the liquid-nitrogen trap. The pressure inside the cavity was not measured directly, but an ionization gauge on the tube used for evacuating the cavity indicated a pressure of 10^{-7} mm Hg.

Microwave equipment

Electron cyclotron resonance was detected by transmitting microwave power through the cavity and using a superheterodyne receiver to detect the transmitted power. Absorption of power at cyclotron resonance by the electrons in the cavity caused a decrease in the transmitted power. The magnetic field at the cavity was modulated at 70 c/s over a small fraction of the width of the electron cyclotron resonance, and the resulting 70 c/s signal which appeared at the superheterodyne detector was fed to a phase-sensitive detector locked to the 70 c/s modulating oscillator. The resulting signal was, in amplitude and sign, the derivative of the electron cyclotron signal.

The klystron used as the source of microwave power was stabilized in terms of a 10 Mc/s crystal oscillator as reference by mixing the klystron signal with the

9720 Mc/s harmonic of the crystal oscillator and detecting a beat near 114 Mc/s with an f.m. communications receiver. The signal from the discriminator of the receiver was used to control the reflector voltage of the klystron. In this way the klystron frequency could be stabilized and measured to better than 0·1 p.p.m. The microwave superheterodyne detector was of conventional design, using a local oscillator with electronic frequency control to obtain a stabilized intermediate frequency of 42·5 Mc/s, a CV 2154 silicon crystal as first detector and a low noise i.f. preamplifier.

Proton spin-resonance measurement

The reference sample of liquid paraffin located in the centre of the top of the microwave cavity envelope was enclosed in a glass cylinder 3·5 mm in diameter and 13 mm long. Round this was wound an r.f. coil connected in a bridge of the type used by Anderson (1949). The bridge was supplied from a stable Clapp type oscillator near 15 Mc/s which was also used to supply the proton resonance magnetic field stabilizer. The difference between this frequency and the 15 Mc/s harmonic of the laboratory frequency standard was measured with a Venner TSA 1035 P counter which registered the number of cycles in the difference frequency which occurred in 1 sec, so that the oscillator frequency could be determined to an accuracy of better than $\pm 0{\cdot}2$ p.p.m.

The determination of ν_e/ν_n requires a knowledge of the proton spin-resonance frequency at the point where the electrons are located which provide the electron cyclotron signal. This point was found by mounting three small coils one above the other with their axes perpendicular to the main magnetic field (which was vertical) and intersecting the axis of the cavity. A small current through one of these coils produced negligible change of total magnetic field (which determines ν_e) at points on the horizontal plane in which its axis lay, but a significant change elsewhere. Switching on the current caused a shift of the position of the observed cyclotron resonance peak except when the electrons which caused the peak were in the same horizontal plane as the axis of the coil. By observing the shift due to each coil in turn the position of the 'effective electrons' could be located to ± 1 mm. The position of these electrons having been found, the value of the difference between the magnetic field strengths at this position and the position of the reference proton sample was required. This was found by removing the cavity assembly from the magnet, replacing the proton resonance reference sample in the gap in the same position as it occupied when mounted on top of the cavity, and comparing the field at this sample with that at two further samples placed vertically below the reference sample and on either side of the estimated position of the effective electrons. The latter two samples were of liquid paraffin enclosed in a thin-walled glass tube 5 mm diameter and 70 mm long, with the r.f. coil wound over a length of 5 mm at the centre. A coil provided local modulation of the magnetic field at the three samples at 70 c/s. By means of a phase sensitive detector locked to 70 c/s the signals from the reference sample and the two plotting samples were in turn set to the peak of the proton spin resonance curve by varying the d.c. current through the stabilizer modulation coils. This current had previously been calibrated in terms of the corresponding shift of the proton resonance frequency, so that the difference in field between the reference sample and the plotting samples could be determined. In this way the magnetic field at the effective electrons could be found in terms of the proton spin-resonance frequency in a long cylindrical sample of liquid paraffin to within ± 4 p.p.m. This figure takes into account the reproducibility of the field distribution before and after a determination and the uncertainty in the positions of the plotting sample and the effective electrons. The reference sample was used only as an intermediary and consequently any shape factor for this sample was irrelevant.

3. Experimental procedure

Before each set of determinations of ν_e the cavity was removed from the gap, baked at 400 °C for at least 24 h, and the filament was raised to a temperature higher than that used during the determination. The magnetic field was adjusted for good homogeneity by the cycling procedure described in the previous paper. The 'field correction' between the reference sample position and the effective electron position was measured as described in the previous section. The cavity was then located in the gap and the microwave frequency was brought to within $\pm 0{\cdot}5$ Mc/s of the peak of the cavity resonance by adjusting the frequency for maximum transmitted power through the cavity, as shown by maximum i.f. level. The two adjustments necessary for obtaining the peaks of both the proton spin-resonance absorption in the reference sample and the electron cyclotron absorption in the cavity were the frequency of the proton resonance Clapp oscillator (close to 15 Mc/s) and the d.c. current through the proton resonance stabilizer head, which also varied the magnetic field strength at the cavity. These adjustments were made concurrently by two operators until the appropriate phase sensitive detectors showed a null reading, that is, when the conditions were correct for obtaining peak absorption in both cases. Care had previously been taken that the proton resonance bridge had been adjusted to give a pure absorption signal. The proton resonance frequency and the microwave frequency were then recorded. Each datum so measured was defined by three parameters, the electron current i_g to the cavity grid, the potential V_g of the grid with respect to the centre of the filament, and the potential V_c of the grid with respect to the upper part of the cavity. A series of datum points taken with various values of i_g as V_g and V_c were held constant constituted a run, and the latter potentials were varied from run to run. After each day's measurements the cavity was removed from the magnet and the field correction was redetermined.

4. Discussion of results

The results may be summarized as follows. When the electron current i_g to the grid is varied by adjusting the filament current under otherwise constant conditions the observed value of the electron cyclotron resonance frequency decreases linearly with increasing i_g in the range investigated from $0{\cdot}4$ to $1{\cdot}6\ \mu$A. The frequency obtained by extrapolating the observed values to $i_g = 0$ is, within the precision of the determinations, found to be independent of the potentials V_g and V_c of the grid and the cavity. The ranges of the applied values of the potentials over which signals were measured were $V_g = 0$ to $+1$ V and $V_c = 0$ to $+1{\cdot}6$ V. Allowing for the contact difference of potential between the tungsten filament and the silver plating of the cavity it appears likely that the actual potential of the grid varied from a small negative value with respect to the filament centre, through zero to a positive value. The cyclotron resonance was interpreted as being due to slow electrons near the grid (as confirmed by the 'effective electron' location method described above). Decreasing i_g reduced the electron density without changing the spatial distribution of the electrons and so extrapolation to $i_g = 0$ removes the shift of the cyclotron resonance caused by the electron space charge. Other shifts are small or negligible.

Before discussing the interpretation of the data in detail it is necessary to consider the relation of the free electron cyclotron resonance frequency ν_e to the observed values, and particularly to consider the perturbation caused by electric fields and relativistic effects. An elementary treatment of the resonant absorption of microwave energy will first be discussed.

The non-relativistic equations of motion of an electron in a rectangular Cartesian co-ordinate system, in which a steady magnetic field H exists in the z-direction, are, in Gaussian units

$$\frac{\mathrm{d}v_x}{\mathrm{d}t} = -\frac{e}{m}\left(E_x + \frac{v_y H}{c}\right),$$

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} = -\frac{e}{m}\left(E_y - \frac{v_x H}{c}\right),$$

$$\frac{\mathrm{d}v_z}{\mathrm{d}t} = -\frac{e}{m}E_z,$$

where v_x, v_y, v_z and E_x, E_y, E_z are the components of velocity and electric field, respectively. The component E_z plays no part in the resonant absorption of energy and its value will be taken to be zero, as will the value of v_z at all times. The effect of a finite value of E_z will be treated later.

Liebes & Franken (1959) show that these equations yield for the energy ΔW gained by an electron which is emitted with a velocity v_0 at $t = 0$ into a region where the magnitude of the electric field $(E_x^2 + E_y^2)^{\frac{1}{2}} = E\sin(\omega t + \phi)$

$$\Delta W = \frac{(eE)^2}{m}\frac{\sin^2\frac{1}{2}(\omega-\omega_e)t}{(\omega-\omega_e)^2} \tag{1}$$

$$+\frac{(eE)^2}{m}\frac{\sin^2\frac{1}{2}(\omega+\omega_e)t}{(\omega+\omega_e)^2} \tag{2}$$

$$+\frac{(eE)^2}{2m}\left\{\frac{\cos 2\phi + \cos 2(\omega t+\phi) - \cos[(\omega-\omega_e)t+2\phi] - \cos[(\omega+\omega_e)t+2\phi]}{(\omega-\omega_e)(\omega+\omega_e)}\right\} \tag{3}$$

$$+v_0 eE\left\{\frac{\cos[(\omega-\omega_e)t+\phi-\theta]-\cos(\phi-\theta)}{(\omega-\omega_e)} + \frac{\cos[(\omega+\omega_e)t+\phi+\theta]-\cos(\phi+\theta)}{(\omega+\omega_e)}\right\}, \tag{4}$$

where $\omega_e = 2\pi\nu_e$ and $\tan\theta = (v_y/v_x)_0$, the ratio of the values of v_y and v_x at $t = 0$.

Considering each term in turn:

(1) is the main resonant term, which has a maximum value for $\omega = \omega_e$.

(2) is an antiresonant term. It causes a shift of the maximum value of the resonance as described by terms (1) and (2) equal to $-3(\delta\omega/\omega_e)^4\omega_e$ where $\delta\omega$ is the

observed width of the resonance, assumed to be caused only by the lifetime of the electrons in the cavity. The value of this shift was typically $-5\times10^{-20}\,\omega_e$ and is thus negligible.

(3) is a term which contains ϕ, the phase angle of the microwave field at the time of emission of the electron. The shift of the observed resonance due to this term is $-3\cos 2\phi(\delta\omega/\omega_e)^2\,\omega_e$ which in the most unfavourable case would equal $10^{-9}\omega_e$. In practice it is likely that there is no correlation between ϕ and the time at which an electron enters the microwave field, so that this term would average to zero.

(4) contains the initial electron velocity $v_0 = (v_x^2+v_y^2)_0^{\frac{1}{2}}$ and contains a resonant and an antiresonant term. The latter is negligible under all conditions which need be considered. The first term averages to zero if the angle between the initial electron velocity and microwave field vectors is random. If, on the other hand, for all electrons $(\phi-\theta)=\pi$ at $t=0$ the resultant shift of the resonance maximum would be $6(W_0/W_g)\,(\delta\omega/\omega_e)\,\omega_e$ where W_0 is the initial electron energy and W_g is the total energy gain from the electric field. Since these two latter quantities might typically be of the same order of magnitude the resulting shift could be as high as 100 p.p.m. There is no reason, however, to suppose that there is any significant correlation between the time at which the electron enters the cavity and the value of the microwave field at that instant.

Shifts due to electric fields

Uniform electrostatic fields do not directly change the frequency of the resonance peak, but the resulting velocity change may cause a relativistic change of mass of the electron, which will be considered later. The shift due to non-uniform fields is given by

$$\frac{\Delta\omega}{\omega_e} = \frac{e}{2m\omega_e^2}\left(\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}\right),$$

where the field gradients are evaluated at the orbit centre and, it is assumed, do not significantly change across the orbit.

The non-uniform fields can arise from the space charge of the electrons themselves, or from the distribution of potential on the cavity walls. The effect of the former was eliminated experimentally by extrapolating the observed resonance frequency to zero electron current, and any shift due to the potential of the cavity walls could then have been detected separately by finding the extrapolated value for various potentials on the two parts of the cavity. The absence, within the precision of the experiment, of any shift in this case leads to the view that the only significant shift was due to the space charge of the electrons themselves.

Relativistic shift

For an electron in a cyclotron orbit the fractional shift of the cyclotron frequency from the value $\omega_e = He/mc$ is equal to $-(T-mc^2)/mc^2$ where T is the total energy of the electron. Thus an energy gain of 1 eV would cause a shift of close to -2 p.p.m. The energy gain due to cyclotron resonance depends upon the magnitude of the microwave electric field, and when it becomes significant it causes an asymmetry of the resonance and a frequency shift of its peak value. These effects were investigated by varying the microwave power to the cavity with a calibrated attenuator and observing the resulting shift of the peak of the resonance curve. Asymmetry of the resonance curve was apparent when a shift of the peak was detected. The result of varying the microwave power in this way on the position of the peak is shown in figure 2. By using for all the determinations of ν_e a relative microwave power of 24 dB the effect of this relativistic shift was reduced to $+0{\cdot}25 \pm 0{\cdot}25$ p.p.m.

The lifetime of the effective electrons in the cavity, deduced from the width of the resonance peaks, was of the order of 10^{-6} s. The effective electrons were confined to a region in the cavity of the order of 1 mm high where their velocity must consequently have been very low. The only alternative to this is the supposition that they

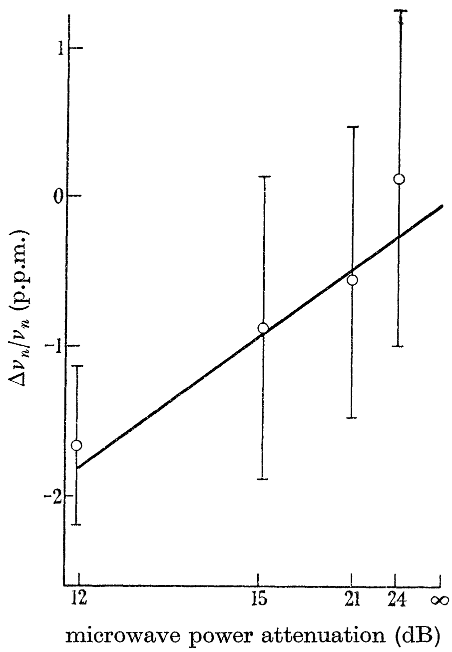

FIGURE 2. The shift with microwave power.

were oscillating in a potential well, but this supposition would be difficult to justify. Ruling out this latter possibility, any shift due to translational motion of the electrons would consequently be that corresponding to an electron energy of much less than 1 eV, and so may be neglected. Similarly the magnitude of the z-component of the microwave electric field necessary to give a significant relativistic shift is far higher than could have existed in the cavity; for a cavity of perfect cylindrical shape the z-component of the electric field is zero in the TE modes, and, because of the very low microwave power used, could in practice have had only a very small value in the cavity used. The relativistic shift due to the trochoidal motion of the electrons caused by a uniform field in the xy plane can also be neglected as a field strength of the order of 10^3 V cm^{-1} would be needed to give a shift of 1 p.p.m.

The electrons are emitted from the filament with a finite energy, the mean value of which is estimated to be $0{\cdot}15 \pm 0{\cdot}05$ eV. Figure 4, illustrating the dependence of the observed value of ν_e on V_g, shows that the z-component of the velocity associated with this initial energy may produce a small effect. The component of velocity in the xy plane would produce a relativistic shift estimated to be $0{\cdot}1 \pm 0{\cdot}05$ p.p.m, which is very small but is also taken into account in the final result.

The effect of cavity tuning

Near cyclotron resonance the electrons in the cavity show a finite susceptibility χ with both real, χ', and imaginary, χ'', components. The latter is responsible for the signal observed on the microwave receiver, and the real part of the susceptibility alters the tuning of the cavity. Provided that the effect is small (which is the case in practice) and that the cavity is tuned exactly to the microwave frequency the variation of χ' with frequency causes no change in the power transmitted through the cavity. If the cavity is not exactly tuned, however, the variation of χ' causes an asymmetry in the observed absorption peak due to the electron resonance and a consequent shift of the peak position. If $f(\omega)$ represents the true absorption $(\mathrm{d}W/\mathrm{d}\omega)$ in the absence of this asymmetry, then the observed absorption peak when the cavity is not tuned exactly to the microwave frequency is (Ward 1961)

$$g(\omega) = f(\omega)\{1 + 2Q_l\,(\omega_e/\delta\omega)\,[(\omega-\omega_c)/\omega]\,[(\omega-\omega_e)/\omega] + \dots\},$$

where Q_l is the loaded Q of the cavity, $\delta\omega/\omega_e$ is the fractional line width of the electron cyclotron resonance, ω_c is the frequency to which the cavity is tuned and $(\omega-\omega_e)$ is the difference between the microwave frequency ω and the electron cyclotron frequency. Under the conditions of these experiments the electron cyclotron resonance is much narrower than the cavity tuning curve (i.e. $(\delta\omega/\omega_e) \ll 1/Q_l$) and the cyclotron resonance peak is shifted by the second term in the bracket. The effect was investigated experimentally, as shown in figure 3, from which it appears that a cavity mistuning of ± 1 Mc/s produces a shift of $\pm 0{\cdot}5$ p.p.m. By adjusting the microwave frequency to give maximum power transmission through the cavity and ensuring that no signal showed more than $0{\cdot}1$ asymmetry (defined as the fractional difference between the maximum values of the phase sensitive detector signals on each side of the resonance) the error due to this cause was reduced to $\pm 0{\cdot}25$ p.p.m.

The effect of magnetic field modulation

Electron cyclotron resonance was observed by modulating the magnetic field at the cavity at 70 c/s and detecting the 70 c/s signal from the microwave receiver. The effect of the finite amplitude of the modulation would be to broaden a symmetrical resonance line, and to shift the peak of an asymmetrical line. The modulation amplitude used throughout the determinations was about $0{\cdot}2$ of the resonance line-width, and no significant broadening of the line was observed until the amplitude was increased until it became comparable with the line width. Since the lines used in the determination of ν_e invariably showed insignificant asymmetry, no error from this source need be considered.

The shape of the resonance line

In order to predict the observed shape of the electron cyclotron absorption curve a lifetime distribution function for the electrons must be postulated. If N electrons are present in the cavity at any time, and $L(t)$ is the fraction having a lifetime in the range t to $t+\mathrm{d}t$ then the power absorbed is

$$P = N\int_0^\infty \Delta W L(t)\,\mathrm{d}t.$$

The function $L(t)$ which gives very satisfactory agreement with the observed peak shape is that mathematically equivalent to gas collision broadening, i.e.

$$L(t) = \mathrm{e}^{-(t/\tau)}/\tau,$$

which leads to a line of Lorentzian shape, where τ is a characteristic lifetime,

$$P = \frac{N(eE)^2}{4m}\frac{\tau^2}{1+(\omega-\omega_e)^2\tau^2}.$$

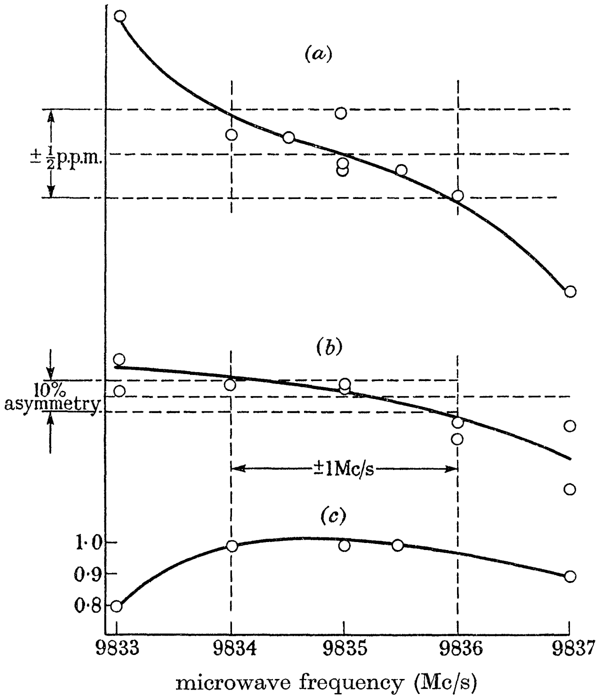

FIGURE 3. The effect of cavity mistuning. (a) Fractional variation in ν_p; (b) signal asymmetry; (c) relative i.f. level.

The magnetic field due to the filament current

The current through the filament produced at the site of the effective electrons a magnetic field of almost 1 G. At a point immediately above the centre of the filament (we take the field of the magnet as being vertical) this field was horizontal and caused an increase of the total field strength of only about 0·05 p.p.m. At a point immediately above the outer edge of the filament the z-component of the field due to the filament could have increased the total field by about 5 p.p.m. The effect was investigated experimentally by heating the filament either with d.c. or with 10 kc/s a.c. In both cases the breadth of the electron cyclotron signal obtained, keeping all other parameters fixed, was the same, i.e. 20 ± 2 p.p.m., and the position of the peak of the signal was shifted by 3 ± 3 p.p.m. when d.c. was used, compared with its position when the filament was heated with a.c. The direction of this shift was the same irrespective of the direction of the d.c. From these results it is concluded that there was no significant effect due to a z-component of filament field, and that, using a.c. heating, the shift in the peak position could have been no greater than 1 ± 1 p.p.m. Because of the construction of the cavity in the vicinity of the filament it is likely that, using a.c. heating, the interior of the cavity was shielded from the magnetic field of the filament by the induced eddy currents.

The filament was operated typically with a current of 1·5 A r.m.s. at 10 kc/s, which produced a peak potential between the ends of the filament of 2·3 V.

5. The value of ν_n

The higher precision of this measurement compared with that described in the preceding paper necessitates further consideration of the corrections to the value of ν_n due to the shape of the sample used. The plotting samples used as the final standards for determining ν_n were long glass cylinders filled with liquid paraffin. Dickinson (1951) considers the circumstances under which the magnetic field $(H + \delta H)$ experienced by a proton may differ from the applied field H. Effects which are relevant in the case of a long cylindrical oil sample with its axis perpendicular to the field are:

(i) diamagnetic shielding by the electrons surrounding the proton;
(ii) second order paramagnetism (in polyatomic molecules);
(iii) bulk diamagnetism of the sample;
(iv) static nuclear paramagnetic fields;
(v) Bloch–Siegert shift due to the antiresonant component of the applied linear r.f. field.

Of these the last two are negligible, both being less than 0·01 p.p.m. in the present determination. Corrections for (i) and (ii) have been calculated for molecular hydrogen by Ramsey (1950) and by Newell (1950) to be

$$\delta H/H = -26{\cdot}6 \pm 0{\cdot}3 \text{ p.p.m.}$$

Experimental comparison of molecular hydrogen with liquid paraffin (mineral oil) has been carried out by Thomas (1950) who finds that for the latter

$$\delta H/H = -28{\cdot}2 \pm 0{\cdot}4 \text{ p.p.m.}$$

The bulk diamagnetism correction (iii) for an infinitely long transverse cylinder is, in c.g.s. units

$$\delta H/H = (4\pi/3 - 2\pi)\,\chi_v,$$

where χ_v for liquid paraffin is taken as $-0{\cdot}65 \pm 0{\cdot}05 \times 10^{-6}\,\text{cm}^{-3}$, giving

$$\delta H/H = 1{\cdot}36 \pm 0{\cdot}10 \text{ p.p.m.}$$

Thus the correction from the experimental result using a long cylindrical liquid paraffin sample to the result appropriate to free protons is $-29{\cdot}6 \pm 0{\cdot}4$ p.p.m.

6. Experimental results

The evidence that the observed value of ν_e is independent of the potentials V_c and V_g is shown in figures 4 to 6, where, effectively, the values of ν_e, extrapolated to $i_g = 0$, are plotted against V_g and V_c. The ordinate is the value of the quantity

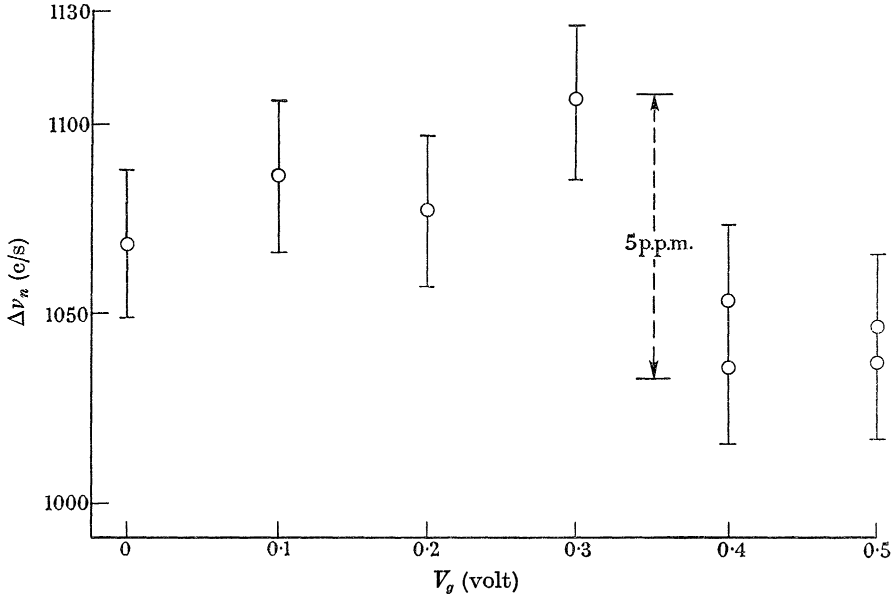

Figure 4. The effect of the variation of grid-filament potential, run IV.

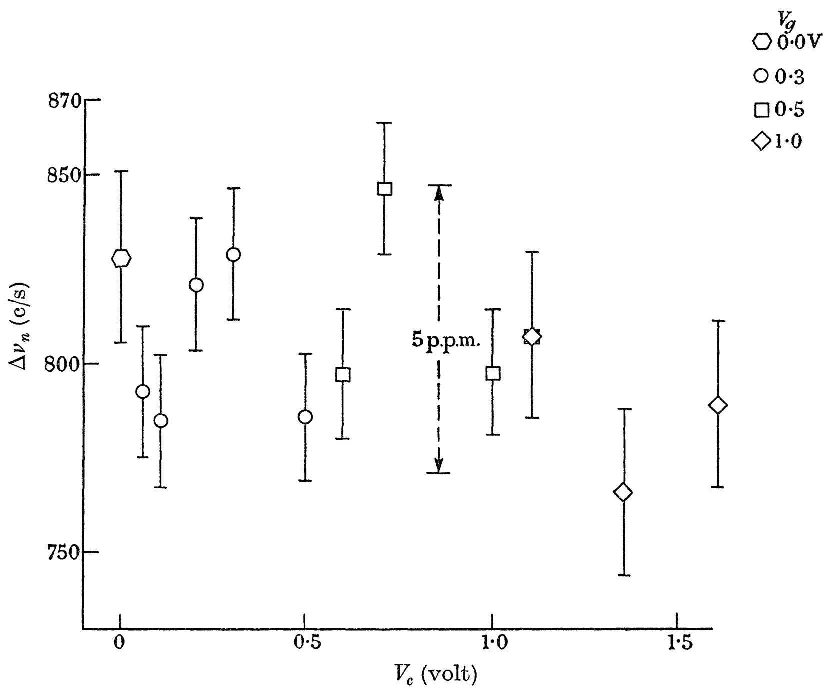

Figure 5. The effect of the variation of grid-cavity potential, run I.

$\Delta\nu_n = 14960000 - \nu_n$ c/s where ν_n is the value of the proton resonance frequency at the reference sample when the electron cyclotron resonance signal has its peak value. All the points shown on one graph were taken on the same day so that the same magnetic field correction applies to all these points, apart from any possible time variation of this correction which is estimated to be not more than ± 1 p.p.m.

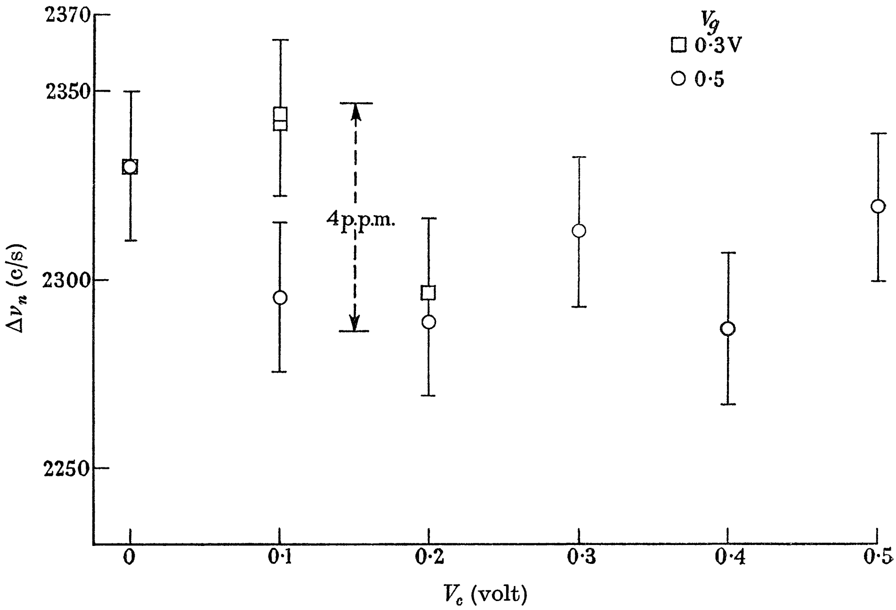

FIGURE 6. The effect of the variation of grid-cavity potential, run II.

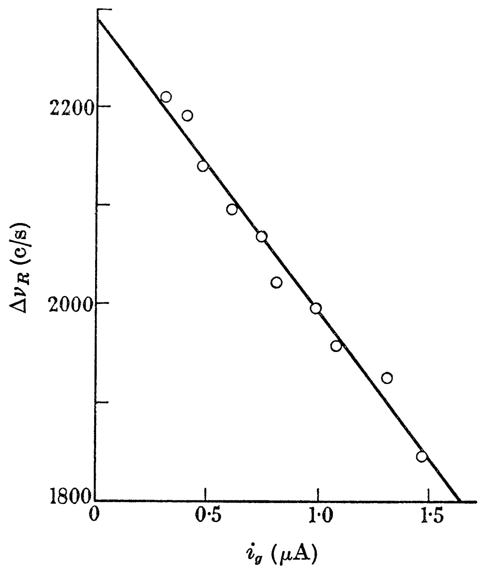

FIGURE 7. Typical plot showing linear extrapolation to $i_g = 0$, run II 7. Intercept, 2290 ± 8 c/s; slope, -295 ± 9 c/s $\div$ μA.

A typical plot of $\Delta\nu_n$ against the current i_g is shown in figure 7. The error quoted in the position of the intercept is obtained from a straight line least squares fit to the experimental points. In table 1 is shown the extrapolated values of $\Delta\nu_n$ obtained in this way. The values of the microwave frequency used in each of the runs referred to in table 1 are shown, with the relevant data, in table 2. The derived values of ν_e/ν_n are

TABLE 1. CYCLOTRON RESONANCE DATA

run no.		V_g (V)	V_c (V)	$\Delta\nu_n$ ($i_g = 0$) (c/s)	slope of $\Delta\nu_n$ *vs.* i_g (c/s ÷ μA)	ν_e/ν_n	probable error in ν_e/ν_n
I	1	1·0	1·6	790 ± 15	211 ± 18	657·4601	± 0·0025
	2	1·0	1·35	767 ± 16	136 ± 13	657·4591	25
	3	1·0	1·1	808 ± 14	167 ± 13	657·4609	25
	4	0·5	1·0	798 ± 8	332 ± 9	657·4605	25
	5	0·5	0·7	847 ± 5	251 ± 4	657·4626	24
	6	0·5	0·6	797 ± 7	207 ± 7	657·4604	24
	7	0·3	0·5	786 ± 4	242 ± 4	657·4600	24
	8	0·3	0·3	829 ± 7	302 ± 7	657·4618	24
	9	0·3	0·2	821 ± 5	282 ± 7	657·4615	24
	10	0·3	0·1	785 ± 6	268 ± 7	657·4599	24
	11	0·3	0·05	793 ± 7	265 ± 7	657·4602	24
	12	0·0	0·0	828 ± 17	414 ± 20	657·4618	25
II	1	0·5	0·0	2329 ± 11	341 ± 15	657·4634	± 0·0027
	2	0·3	0·2	2297 ± 7	280 ± 9	657·4620	27
	3	0·3	0·1	2343 ± 7	280 ± 9	657·4640	27
	4	0·3	0·1	2342 ± 5	290 ± 4	657·4640	26
	5	0·3	0·0	2330 ± 12	292 ± 11	657·4635	27
	6	0·5	0·3	2313 ± 6	301 ± 7	657·4627	26
	7*	0·5	0·2	2290 ± 8	295 ± 9	657·4617	27
	8	0·5	0·1	2295 ± 9	284 ± 9	657·4620	27
	9	0·5	0·4	2288 ± 3	280 ± 4	657·4616	26
	10	0·5	0·5	2320 ± 4	290 ± 4	657·4630	26
III	1	0·5	0·5	784 ± 20	139 ± 20	657·4629	± 0·0026
IV	1	0·5	0·2	1037 ± 9	233 ± 9	657·4650	± 0·0045
	2	0·4	0·2	1036 ± 11	231 ± 11	657·4650	45
	3	0·3	0·2	1106 ± 10	275 ± 9	657·4680	45
	4	0·2	0·2	1077 ± 8	306 ± 9	657·4668	45
	5	0·1	0·2	1086 ± 11	330 ± 11	657·4672	45
	6	0·0	0·2	1068 ± 10	324 ± 9	657·4664	45
	7	0·4	0·2	1053 ± 12	246 ± 11	657·4657	45
	8	0·5	0·2	1046 ± 4	244 ± 4	657·4654	45

* Figure 7.

TABLE 2

run	ν_e (Mc/s)	magnetic field correction (c/s)	interval since cavity last baked (h)	duration of baking (h)
I	9835·000	128 ± 55	24	24
II	9834·000	185 ± 60	96	24
III	9835·000	198 ± 55	3	48
IV	9834·900	144 ± 102	48	48

shown in table 1. The errors in ν_e/ν_n shown in this table are those associated with the uncertainty in the magnetic field correction and a relatively small contribution from the extrapolation procedure. The average result derived from run IV is somewhat higher than that of the other runs, which are in good agreement, but the uncertainty in the magnetic field correction was higher for run IV and no particular significance can be attached to this deviation.

The weighted mean of the thirty-one extrapolated values of ν_e/ν_n shown in table 1 is

$$(\nu_e/\nu_n) = 657{\cdot}4621.$$

The uncertainties associated with this value are:

field correction uncertainty and (almost negligible) extrapolation uncertainty: ± 2 p.p.m.;

possible magnetic contamination of resonator, etc.: ± 1 p.p.m.;

possible dependence of ν_e/ν_n on V_g: $\pm 2{\cdot}5$ p.p.m.;

effect of cavity mistuning: $\pm 0{\cdot}25$ p.p.m.;

corrections applied to the observed value of ν_e/ν_n are:

effect of filament current: $\pm 1 \pm 1$ p.p.m.;

microwave power (relativistic) shift: $0{\cdot}25 \pm 0{\cdot}25$ p.p.m.;

initial electron energy (relativistic) shift: $0{\cdot}1 \pm 0\ 2$ p.p.m.

When these above errors and corrections, and also the correction for the proton sample discussed in § 5 have been taken into account, the final result may be quoted as:

$$\begin{aligned} \nu_e/\nu_n &= 657{\cdot}4630 \pm 0{\cdot}0024(\pm 3{\cdot}6\,\text{p.p.m.})\ \text{(long cylindrical liquid paraffin sample)} \\ &= 657{\cdot}4620 \pm 0{\cdot}0024(\pm 3{\cdot}6\,\text{p.p.m.})\ \text{(spherical liquid paraffin sample)} \\ &= 657{\cdot}4436 \pm 0{\cdot}0025(\pm 3{\cdot}8\,\text{p.p.m.})\ \text{(free protons).} \end{aligned}$$

The final uncertainty has been obtained by assuming that the individual uncertainties quoted are probable errors.

Earlier published results for this quantity are:

Gardner (1951) $657{\cdot}475 \pm 0{\cdot}008$ (spherical oil sample);

Liebes & Franken (1959) $657{\cdot}462 \pm 0{\cdot}003$ (spherical oil sample);

DuMond (1959) quotes an unpublished result by Hardy of $657{\cdot}4676 \pm 0{\cdot}0010$ (spherical oil sample).

The magnetic moment of the proton derived from the result of the work described here is

$$(1{\cdot}521043 \pm 0{\cdot}000006) \times 10^{-4}\ \text{Bohr magneton.}$$

The authors are indebted to Professor B. Bleaney, F.R.S., for his encouragement and for providing facilities which made the work possible. They are also grateful to Mr R. W. Huggins and Mr C. E. Webb for their assistance in making experimental observations. One of us (J. F. W.) expresses his gratitude to the Department of Scientific and Industrial Research for financial support and another (K. F. T.) to the Scott Fund for the award of a scholarship.

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