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Feedback Controlled Self-Excitation of Optical Pulses in a CW Dye Ring Laser*

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Abstract. Feedback controlled self-excitation of optical pulses of nanosecond duration has been observed to occur rather reliably in a CW dye ring laser, oscillating in a traveling mode. These observations are described analytically by means of a direct time domain approach. It is shown that a steady-state Gaussian pulse whose time duration determined from the self consistency condition in terms of system characteristics describes accurately the observed pulse behaviors.

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There is considerable theoretical and experimental interest in the generation of optical pulses covering both picosecond and nanosecond time regimes. In particular, short time duration dye laser pulses have proven to be a powerful tool in time dependent spectroscopic applications. The time domain analysis of steady state operating characteristics of both actively and passively mode-locked pulses have been developed in recent years [1–5]. Feedback controlled self-excited optical pulses of nanosecond duration have been observed from a CW dye ring laser oscillating in a traveling mode [6]. This kind of pulse generation mechanism differs from the previous self-excitation of optical pulses [7, 8] in so far that an extra cavity element feeds back a fraction of one of the two traveling waves into the cavity and thereby triggers the self-excitation. This scheme of pulse generation is useful because of its simplicity, reliability, and independence from the pulse carrier frequency.

The purpose of this paper is to present a time-domain description of this type of feedback controlled optical pulse generation. The theoretical model starts from the usual rate equation for two waves traveling in the opposite directions. It takes into account the periodic spatial modulation of the gain factor induced and concomitant coupling of two waves, and finally the superposition of two waves due to the feedback arrangement. It is shown that the steady state pulse is a Gaussian pulse whose time duration is determined by the cavity round-trip time, the operating power level, and the relative phase at which these two pulses are phase locked.

1. Experimental Observations

A brief description of the experimental observations on traveling-wave oscillations in a CW dye ring laser [6] is presented. The ring-laser configuration, together with various quantities of interest considered in the theory are shown in Fig. 1. The laser consists of an Ar^+ laser pumped jet of rhodamine 6G of active length L, with $L \ll cT$, c, T, being the velocity of light and the cavity round trip time, respectively. The typical round trip time T ranges from 5 to 15 ns. Let V_+, V_- denote, respectively, the wave envelope traveling in the counter clockwise and clockwise direction. A fraction of the wave V_- is fed back into the cavity by means of mirror M_1 and is superposed to the wave V_+. The laser can be wavelength tuned by the counter rotation of mirror M_2 and M_3, using the dispersive prisms P_1 and P_2. Attached to prism P_1 is P_1^*, which serves as an out-coupler of adjustable transmission.

The characteristics of the self-excited optical pulses may be summarized as follows [6]: (i) typical pulse

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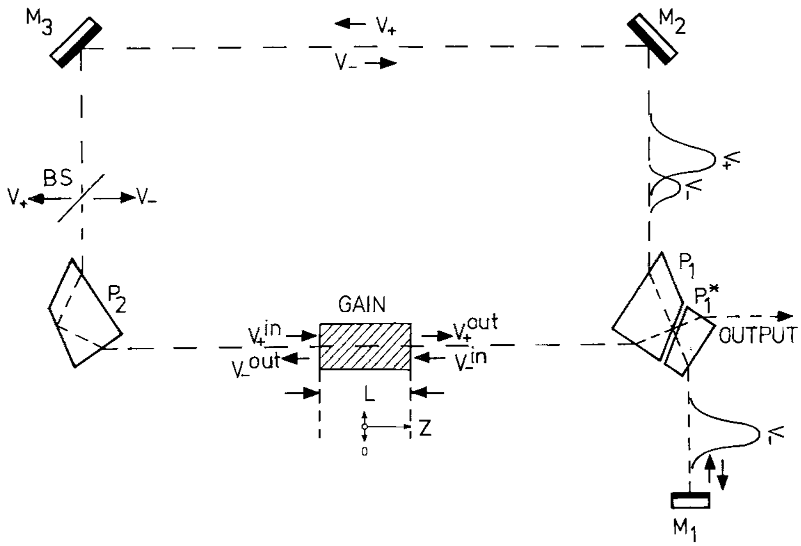

Fig. 1. Feedback controlled mode-locking configuration. Mirror M_1 converts a fraction of V_- into V_+

duration of a few ns can be generated reliably with the mirror M_1 well aligned and for laser bandwidth of a few Å. (ii) The carrier frequency of the pulse can be tuned over the entire working range of rhodamine 6G. (iii) The intensity ratio, $|V_+|^2/|V_-|^2$ is always greater than 10, but $|V_-|^2$ never goes to zero. This ratio was monitored by the beam splitter (BS) inserted in the ring cavity. (iv) The pulsing behavior starts at the laser threshold and vanishes at extremely high optical pumping levels above threshold. In fact, the self-pulsing mode is more stable than CW mode. (v) The stability criteria as to the multiple pulses are similar to picosecond mode-locking behavior [2–5], for which at higher pumping levels and long cavities multiple pulses are more stable than a single pulse. The cavity round trip time is greater than the life time of the excited lasing level of rhodamine 6G(~ 5 ns).

2. Theory

The laser field inside the ring cavity configuration consists of two waves traveling in the opposite directions:

$$E(z,t) = \tfrac{1}{2}[V_+(z,t)e^{i\Psi_+} + V_-(z,t)e^{i\Psi_-} + \text{c.c.}], \tag{1}$$

$$\Psi_\pm(z,t) = \omega_0 t \mp k_0 z, \tag{2}$$

$$V_\pm(z,t) = \int d(\omega-\omega_0)\tilde{E}_\pm(\omega-\omega_0, z)e^{i(\omega-\omega_0)t}. \tag{3}$$

Here, the carrier frequency ω_0 is taken to be equal to the atomic transition frequency for simplicity (a non-essential approximation). The response of the medium caused by these waves can be obtained by using the usual rate equation approach [1, 9, 10]. Because the laser field consists of two counter-running waves, it induces the spatially modulated response in the gain medium. The modulation will then give rise to the energy exchange between the two pulses, while being amplified in the medium.

The effective gain equation of the field components in the medium are given by (see Appendix A).

$$L\frac{\partial \tilde{E}_+}{\partial z} = G(\omega)\tilde{E}_+ - \varkappa \tilde{E}_-$$
$$L\frac{\partial \tilde{E}_-}{\partial z} = -G(\omega)\tilde{E}_- + \varkappa^* \tilde{E}_+, \tag{4}$$

where the dispersive saturated gain and coupling per pass are

$$G(\omega) = G_0\left(1 + 2i\frac{\omega-\omega_0}{\Delta\omega}\right)^{-1}\left(1 + \frac{1}{I_s}\langle |V_+|^2 + |V_-|^2\rangle\right)^{-1}, \tag{5a}$$

$$\varkappa = G_0\langle V_+ V_-^*\rangle / I_s, \tag{5b}$$

$$G_0 = \tfrac{1}{2}(N_e T_2|\mu|^2/3\varepsilon_0\hbar)(k_0/n^2)L, \tag{5c}$$

$$I_s = 3\hbar^2/|\mu|^2 T_1 T_2. \tag{5d}$$

Here, μ, T_1, n, N_e denote the dipole matrix element, the lifetime of the lasing level, the index of refraction of the medium, the population inversion with no applied field, respectively, and the linewidth is $\Delta\omega = 2/T_2$. The angular bracket represents the time average over the pulse repetition period. L is the length of the gain medium. The above model for gain applies to (a) a homogeneously broadened system near threshold, (b) the pulse bandwidth much smaller than the atomic linewidth, and (c) the case where the time dependence, $\Delta N(t)$ of the population inversion due to the laser field is to be neglected. Since the medium consists of a dye jet flowing perpendicular to the cavity axis, the Doppler broadening is not important. In addition, the time duration of the generated pulses is typically a few nanoseconds, whereas the atomic bandwidth is ~ 1 Å. Finally $\Delta N(t)$ becomes negligible when either T_1 is much shorter than the pulse duration or the operating power level of the pulse is not high enough to cause appreciable saturation during the pulse duration. Note that the coupling strength, $\varkappa$ is determined by the steady power level of the two pulses.

The exact solution of (4) can be found readily. However, by using a rather practical approximation, viz. the single pass gain is small for $\tilde{E}_+, \tilde{E}_-$ entering the medium at opposite ends, the respective output can be simplified as (Appendix B)

$$\begin{bmatrix}\tilde{E}_+\\ \tilde{E}_-\end{bmatrix}_{\text{out}} = \begin{bmatrix}1+G(\omega) & -\varkappa\\ -\varkappa^* & 1+G(\omega)\end{bmatrix}\begin{bmatrix}\tilde{E}_+\\ \tilde{E}_-\end{bmatrix}_{\text{in}}. \tag{6}$$

The steady state characteristics of generated pulses can be described by the time domain analysis [1]. At the outset it is assumed that the two pulses are Gaussian having different amplitudes and phases. The input pulse propagating in the counter-clockwise direction,

$$V_+^{(\mathrm{I})}(t) = \tfrac{1}{2}E_0 \exp(-\gamma t^2), \tag{7a}$$

$$\tilde{E}_+^{(\mathrm{I})} = \tfrac{1}{2}E_0 \sqrt{\pi/\gamma}\exp[-(\omega-\omega_0)^2/4\gamma] \tag{7b}$$

is transformed after passing through the lasing medium as

$$\tilde{E}_+^{(\mathrm{II})} = \mathrm{e}^{G(\omega)-\varkappa r}\tilde{E}_+^{(\mathrm{I})}, \tag{8a}$$

$$V_+^{(\mathrm{II})}(t) = \tfrac{1}{2}E_0 \mathrm{e}^{\bar{G}-\varkappa r}(\Gamma/\gamma)^{1/2}\exp[-\gamma(t-2\bar{G}/\Delta\omega)^2] \tag{8b}$$

with

$$\bar{G} = G_0\Big/\left[1+\frac{\langle |V_+|^2+|V_-|^2\rangle}{I_s}\right], \tag{8c}$$

$$\Gamma = \gamma(1+16\bar{G}\gamma/\Delta\omega^2)^{-1}, \tag{8d}$$

$$r = |\tilde{E}_-^{(\mathrm{I})}|/|\tilde{E}_+^{(\mathrm{I})}|. \tag{8e}$$

Here, the saturated, time averaged gain factor $\bar{G}(\omega)$ has been expanded in terms of $(\omega-\omega_0)/\Delta\omega$ up to the second order, and the effective single pass gain is regarded much less than unity.

Next, the amplified pulse, $V_+^{(\mathrm{II})}(t)$ is linearly superposed with a fraction of $V_-^{(\mathrm{I})}(t)$ due to the feedback element, i.e. the mirror M_1. The resulting pulse envelope is then to be expressed as

$$V_+^{(\mathrm{III})}(t) = V_+^{(\mathrm{II})}(t)(1-Fr\mathrm{e}^{-\mathrm{i}\Delta\phi}) \simeq V_+^{(\mathrm{II})}(t)\exp[-Fr\mathrm{e}^{-\mathrm{i}\Delta\phi}], \tag{9}$$

where F denotes the fraction of $V_-^{(\mathrm{I})}$ fed back into the cavity with its phase reversed at the mirror M_1 and

$$\Delta\phi = \phi_+ - \phi_- \tag{10}$$

is the relative phase between two counter-running waves. The magnitude of the factor, Fr is taken much smaller than unity, as will be shown to be the case a posteriori. It is clear from (9) that the pulse under investigation experiences both amplitude and phase modulation per every cavity round trip time (T). The periodic nature of these modulations in time can be properly incorporated by regarding F as a time varying quantity, i.e. $F=F_0\cos(t/T)$. By assuming that the pulse duration is much shorter than T, one can expand $\cos(t/T)$ with the initial time adjusted to maximize the modulation

$$F(t) \simeq F_0\{1-\tfrac{1}{2}[(t-2\bar{G}/\Delta\omega)/T]^2\}. \tag{11}$$

Upon inserting (11) into (9) there results

$$V_+^{(\mathrm{III})}(t) = \mathrm{e}^{-2\delta+\delta[(t-2\bar{G}/\Delta\omega)/T]^2}V_+^{(\mathrm{II})}(t), \tag{12}$$

where the complex depth of modulation is given by

$$2\delta = F_0 r\mathrm{e}^{-\mathrm{i}\Delta\phi}. \tag{13}$$

The round trip of the pulse is completed by including the cavity loss factor, $\omega_0 T/Q_+$ and the time, L_c/c, L_c being the cavity length. One may write

$$V_+^{(\mathrm{IV})}(t) = \mathrm{e}^{-(\omega_0 T/Q_+)}V_+^{(\mathrm{III})}(t-L_c/c), \tag{14}$$

The self-consistency condition that the input pulse be same as the net output pulse except a possible, constant phase factor, Ψ can be written explicitly as

$$\mathrm{e}^{-\gamma t^2-\mathrm{i}\Psi} = (1+16\bar{G}\gamma/\Delta\omega^2)^{-1/2}\mathrm{e}^{A_+}\mathrm{e}^{(-\Gamma+\delta/T^2)t^2} \tag{15}$$

with

$$A_+ = -\omega_0 T/Q_+ + \bar{G} - \varkappa r - 2\delta. \tag{16}$$

Here, the effective cavity round trip time, $T = L_c/c + 2\bar{G}/\Delta\omega$ including the phase dispersion in the medium has been cancelled from both sides of (15). One can carry out a similar analysis for $V_-(t)$, in which case the net amplitude factor ($\exp A_-$) reads as

$$A_- = -(\omega_0 T/Q_-) + \bar{G} - (\varkappa/r). \tag{17}$$

Equations (15)–(17) describe the operating characteristics of two pulses generated, and are similar in form to the equation derived previously by Kuizenga and Siegman in analyzing AM and FM mode-locking of the homogeneous laser [1]. These equations, however, incorporate different physical processes, in that two waves are always present, amplified and coupled with each other simultaneously in the medium and experience amplitude as well as phase modulation periodically in time induced by the feedback element. The pulse duration is determined from (15), (13), (8d), by the relation,

$$\gamma = [\gamma/(1+16\bar{G}\gamma/\Delta\omega^2)] - (\delta/T^2), \tag{18}$$

and are dependent, among other factors, on the steady state power levels of two pulses ($\bar{G}$) as well as the relative phase, $\Delta\phi$. The respective power level is in turn determined by requiring that the net gain including the cavity loss be unity, i.e. $A_\pm = 0$;

$$-(\omega_0 T/Q_+) + \bar{G} - \varkappa r - 2\delta = 0, \tag{19a}$$

$$-(\omega_0 T/Q_-) + \bar{G} - (\varkappa/r) = 0. \tag{19b}$$

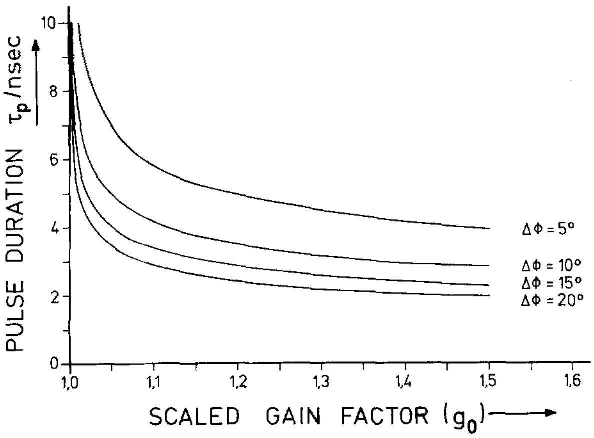

Fig. 2. Pulse duration [ns] as a function of the scaled gain factor for four different relative phase values and for $\Delta\omega = 1$ Å, $T = 10$ ns

Here the cavity loss factors for both waves are taken to be same for simplicity.

In the absence of the feedback element ($\delta = 0$) it becomes clear from (18) and (19) that $r = 1$, $\gamma = 0$. This means that one recovers the CW operation of the ring laser. On the other hand, with a fraction of V_- fed back into the cavity, the ratio of amplitudes, r can be very different from 1. r can be found from (19) as

$$r = \frac{f_0 \cos\Delta\phi}{2g_0\varepsilon}\left\{\left[1 + \left(\frac{2g_0\varepsilon}{f_0\cos\Delta\phi}\right)^2\right]^{1/2} - 1\right\}, \tag{20}$$

with the quantities G_0, F_0 scaled w.r.t. the cavity loss factor,

$$g_0 = G_0/(\omega_0 T/Q)$$

$$f_0 = F_0/(\omega_0 T/Q) \tag{21}$$

and the time averaged power of V_+ is scaled w.r.t. I_s,

$$\varepsilon = \langle |V_+|^2 \rangle / I_s . \tag{22}$$

Near threshold regime where $\varepsilon \ll 1$, r can be much less than unity, provided the relative phase between two pulses, $\Delta\phi$ is not near the value $\pi/2$. The experimental evidence that r is small but remains fairly constant over a rather broad range of pumping power indicates that $\Delta\phi$ may perhaps be dependent on the pumping power and goes to zero with increasing pumping level. In this analysis, $\Delta\phi$ is regarded as a parameter. Near threshold r can be simplified as

$$r \simeq 2g_0\varepsilon / f_0 \cos\Delta\phi . \tag{23}$$

Upon inserting (23) into (19), one obtains the power level of the pulses, i.e.

$$\varepsilon \simeq (g_0 - 1)/(g_0 + 1) . \tag{24}$$

Next, the pulse duration can be discussed explicitly by inserting (23) and (24) into (18). γ is in general a complex quantity: the real part (γ') determines the time duration, while the imaginary part (γ'') gives rise to the constant phase shift, Ψ in (15). For the practical situation where $\Delta\omega T \gg 1$, ($\Delta\omega \sim 1$ Å, $T \sim 10$ ns) one finds for $|\Delta\phi| < \pi/2$

$$\gamma' = \frac{1}{4}\sin|\Delta\phi|\left(\frac{\Delta\omega}{T}\right)\left(\frac{\varepsilon(1+\varepsilon)}{2\cos\Delta\phi}\right)^{1/2} . \tag{25}$$

Equation (25) gives the pulse width, $\tau_p^2 = 1/\gamma'$ in terms of the net cavity round trip time T, the bandwidth $\Delta\omega$, the relative phase $\Delta\phi$, and the scaled gain factor g_0 or equivalently the operating power level ε of the pulse itself. In Fig. 2, τ_p is plotted as a function of gain factor for four different values of $\Delta\phi$ with $\Delta\omega = 1$ Å, $T = 10$ ns, corresponding to a typical experimental situation. As can be noted clearly from Fig. 2, these theoretical values of τ_p can explain the typical observed values of the pulse duration. The dependence of τ_p on g_0 and $\Delta\phi$ is most pronounced near threshold regime. The experimental observation that τ_p is essentially constant over a broad range of g_0 provides an additional evidence that $\Delta\phi$ may perhaps be dependent on g_0, viz. ε. Inversely, assuming τ_p independent of g_0, one can infer the g_0-dependence of $\Delta\phi$ from (25). It appears that the rigorous discussion of $\Delta\phi$ is a formidable, yet interesting problem.

Conclusion

In conclusion, this paper presents a time domain analysis of feedback controlled pulse generation in traveling ring-laser configuration. It has been shown that a Gaussian pulse, whose time duration determined from the self-consistency condition in terms of various system characteristics can explain realistically the observed pulses, in particular the time duration. It has also been pointed out that the role of the feedback element is to be modeled essentially as an amplitude modulator. A further analysis is under way involving the stability criteria of the pulses and a detailed discussion of the relative phase, at which value the two pulses are self-phase locked.

Appendix A

Gain and Coupling of two Counter-Running Waves in a Homogeneous Medium

In this appendix, the effective gain and coupling of two waves traveling in an opposite direction in a homogeneous medium is discussed. The rate equation reads as

$$\left(\frac{\partial^2}{\partial t^2} + \frac{2}{T_2}\frac{\partial}{\partial t} + \omega_0^2\right)P = -\frac{2\omega_0|\mu|^2}{3\hbar}NE \tag{A.1}$$

$$\frac{\partial}{\partial t}N + \frac{N - N_e}{T_1} = \frac{2}{\hbar\omega_0}E \cdot \frac{\partial P}{\partial t}, \tag{A.2}$$

where N, P denote, respectively, the population inversion and the macroscopic polarization and $2/T_2 = \Delta\omega$ is the resultant linewidth including the effects of prisms used. In the two limits where the pulse duration is much shorter than T_1 or the steady state power level not high enough to cause appreciable saturation during the pulse duration, the first term on the left of (A.2) can be neglected thus, one can rewrite (A.2) as

$$N = N_e + \frac{2T_1}{\hbar\omega_0}\frac{1}{T}\int dt\, E\cdot\frac{\partial P}{\partial t}, \quad (A.3)$$

where T is the pulse repetition or round trip time. By regarding N time independent, one finds the susceptibility associated with $\tilde{E}_+(\omega)\tilde{E}_-(\omega)$ as

$$\chi_\pm(\omega) = -\frac{2\omega_0|\mu|^2}{3\hbar\varepsilon_0}N\frac{1}{(\omega_0^2-\omega^2)+\mathrm{i}\Delta\omega\omega}. \quad (A.4)$$

Upon inserting (A.4) into (A.3), and making use of the fact that the pulse bandwidth is much smaller than $\Delta\omega$, i.e. $(\omega_0-\omega)/\Delta\omega \ll 1$, one finds the expression of the saturated population inversion as

$$N = N_e\bigg/\bigg(1+\frac{1}{I_s}\langle|V_+|^2+|V_-|^2 + V_+V_-^*\mathrm{e}^{-2\mathrm{i}k_0z} + V_+^*V_-\,\mathrm{e}^{2\mathrm{i}k_0z}\rangle\bigg) \quad (A.5)$$

with I_s given in the text and angular bracket denoting the time average. Because of the presence of two counter-running waves, N is spatially modulated. The resulting response of the medium can thus be obtained from (A.4) and (A.5).

Near the threshold pumping power, where $\langle|V_\pm|^2\rangle/I_s \ll 1$, one may write

$$\chi(\omega) = \mathrm{i}\chi_{\mathrm{d.c.}}(\omega) - \mathrm{i}(\chi_m\mathrm{e}^{2\mathrm{i}k_0z} + \mathrm{c.c.}) \quad (A.6)$$

with

$$\chi_{\mathrm{d.c.}}(\omega) = \chi_0\bigg/\left(1+2\mathrm{i}\frac{\omega-\omega_0}{\Delta\omega}\right)\left(1+\frac{1}{I_s}\langle|V_+|^2+|V_-|^2\rangle\right), \quad (A.7)$$

$$\chi_m = \chi_0(\langle V_+^*V_-\rangle/I_s), \quad (A.8)$$

$$\chi_0 = |\mu|^2 N_e T_2/3\hbar\varepsilon_0. \quad (A.9)$$

Finally, by substituting (A.6)–(A.9) into the wave equation,

$$\left[\frac{\partial^2}{\partial z^2}+\left(\frac{n}{c}\right)^2\frac{\partial^2}{\partial t^2}\right]E = -\mu_0\frac{\partial^2}{\partial t^2}P$$

with the identity, $P(\omega) = \varepsilon_0\chi(\omega)\tilde{E}(\omega)$ and singling out the respective components, one can obtain (5) in the text.

Appendix B

Amplification of Two Coupled Waves

In this appendix, the amplification and concomitant coupling of two waves $\tilde{E}_+$, $\tilde{E}_-$ in the medium are discussed. Introducing the dimensionless distance $\xi = z/L$, one may rewrite (4) in the text as

$$\frac{\partial}{\partial\xi}\tilde{E}_+ = G(\omega)\tilde{E}_+ - \varkappa\tilde{E}_-, \quad (B.1)$$

$$\frac{\partial}{\partial\xi}\tilde{E}_- = -G(\omega)\tilde{E}_- + \varkappa^*E_+. \quad (B\text{-}2)$$

The solution of this equation can be found in the form

$$\tilde{E}_+ = a_1\mathrm{e}^{\gamma_1\xi} + a_2\mathrm{e}^{\gamma_2\xi}, \quad (B.3)$$

$$\tilde{E}_- = b_1\mathrm{e}^{\gamma_1\xi} + b_2\mathrm{e}^{\gamma_2\xi}. \quad (B.4)$$

Upon inserting (B.3), (B.4) into (B.1), (B.2), there results

$$\gamma_i a_i = G(\omega)a_i - \varkappa b_i, \quad (B.5)$$

$$\gamma_i b_i = -G(\omega)b_i + \varkappa^* a_i \quad (B.6)$$

with $i = 1, 2$. From the secular equation associated with (B.5), (B.6), one finds

$$\gamma_{1,2} = \pm[G(\omega)^2 - |\varkappa|^2]^{1/2}. \quad (B.7)$$

Next, the constants, a_i and b_i are to be found from (B.6) and the boundary condition;

$$\tilde{E}_+^{(\mathrm{in})} = a_1\mathrm{e}^{-1/2\gamma_1} + a_2\mathrm{e}^{1/2\gamma_1}, \quad (B.8)$$

$$\tilde{E}_-^{(\mathrm{in})} = a_1\frac{\varkappa^*}{G(\omega)+\gamma_1}\mathrm{e}^{1/2\gamma_1} + a_2\frac{\varkappa^*}{G(\omega)-\gamma_1}\mathrm{e}^{-1/2\gamma_1}. \quad (B.9)$$

After finding a_1, a_2 and inserting the expressions into (B.3), (B.4), the two output waves are given in terms of inputs as

$$\begin{bmatrix}\tilde{E}_+\\ \tilde{E}_-\end{bmatrix}_{\mathrm{out}} = \begin{bmatrix}M_{11} & -M_{12}\\ -M_{21} & M_{22}\end{bmatrix}\begin{bmatrix}\tilde{E}_+\\ \tilde{E}_-\end{bmatrix}_{\mathrm{in}} \quad (B.10)$$

with

$$M_{11} = M_{22} = \gamma_1/[\gamma_1\cosh\gamma_1 - G(\omega)\sinh\gamma_1], \quad (B.11)$$

$$M_{12} = M_{21}^* = \varkappa\sinh\gamma_1/[\gamma_1\cosh\gamma_1 - G(\omega)\sinh\gamma_1]. \quad (B.12)$$

In the limit where the single pass gain and coupling are small, the matrix elements reduce to the expressions given in the text.

References

1. D.J.Kuizenga, A.E.Siegman: IEEE J. QE-**6**, 694 (1970)
2. H.A.Haus: J. Appl. Phys. **46**, 3049 (1975)
3. H.A.Haus: IEEE J. QE-**11**, 736 (1975)
4. H.A.Haus: IEEE J. QE-**12**, 169 (1976)
5. S.L.Shapiro (Ed.): *Ultrashort Light Pulses*, Top. Appl. Phys., Vol. 18 (Springer, Berlin, Heidelberg, New York 1977)
6. G.Marowsky, R.Cubeddu: J. Appl. Phys. **47**, 5470 (1976)
7. P.W.Smith: IEEE J. QE-**3**, 627 (1967)
8. C.L.Tang, M.Statz, G.DeMars: J. Appl. Phys. **34**, 2289 (1963)
9. A.Iesevgi, W.E.Lamb, Jr.: Phys. Rev. **185**, 517 (1969)
10. M.Sargent, III.: Appl. Phys. **9**, 127 (1976)